STRUCTURES

Bending Moments

CP1843

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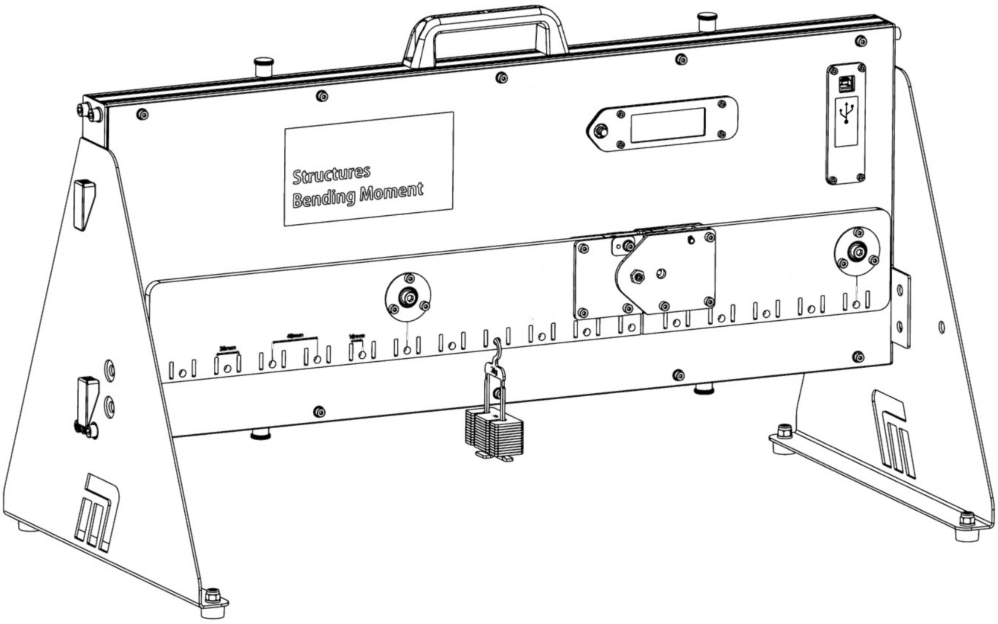
Contents

Bending Moments

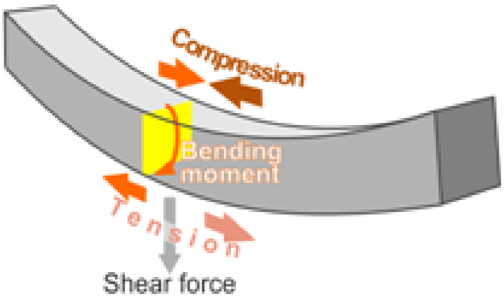
Introduction



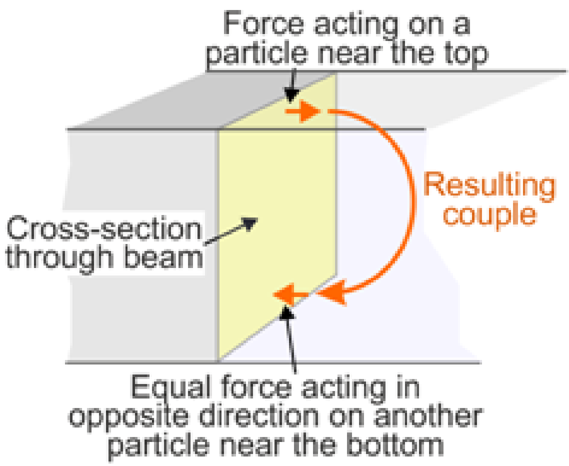
Introduction

Background:

When a force, even its own weight, acts on a structure such as a beam, it can make it bend. Then, the particles making up the beam are squashed closer together near the top surface and stretched further apart near the lower edge.



Internally, the attractive forces between these particles, viewed at a cross-section cut through it, can be considered as a combination of a resultant force and a resultant couple. This resultant internal couple is called the **bending moment**. The resultant internal force is called the **shear force**.



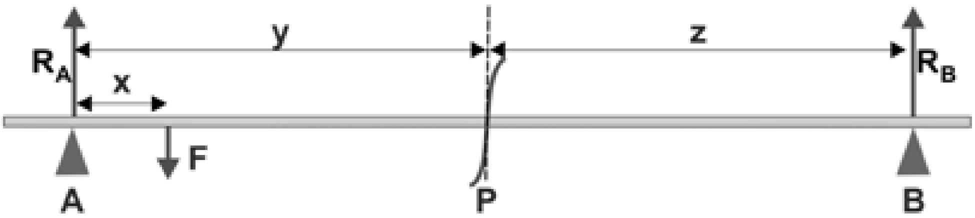
The second diagram illustrates the idea of a bending moment. Looking at a cross-section through the beam, the compressive force on a particle somewhere near the top is the same but opposite to the tensile force on a particle near the bottom. Together they form a couple trying to distort the beam. Adding together the effects of all such particles results in a bending moment.

Background .....

The diagram below illustrates the forces acting on a beam resting on simple supports, **A** and **B**, with a point load **F** acting on it. The supports exert reaction forces R_A and R_B.

The section labelled **AP** is in equilibrium and so:

- the forces acting on it must cancel out;
- and the moments trying to rotate it must cancel out.



Seen at the slice **P**, then, the sum of the clockwise moments is:

$$R_A \times y - F \times (y-x)$$

As **AP** is in equilibrium, there must be an equal opposing anticlockwise moment at the slice. This is the bending moment at **P**.

Hence, the bending moment at **P** = $(R_A \times y) - (F \times (y-x))$

The same argument could be applied from the point of view of the moment of the force R_B at support **B**.

It would give the equation:

Bending moment at **P** = $(R_B \times z)$

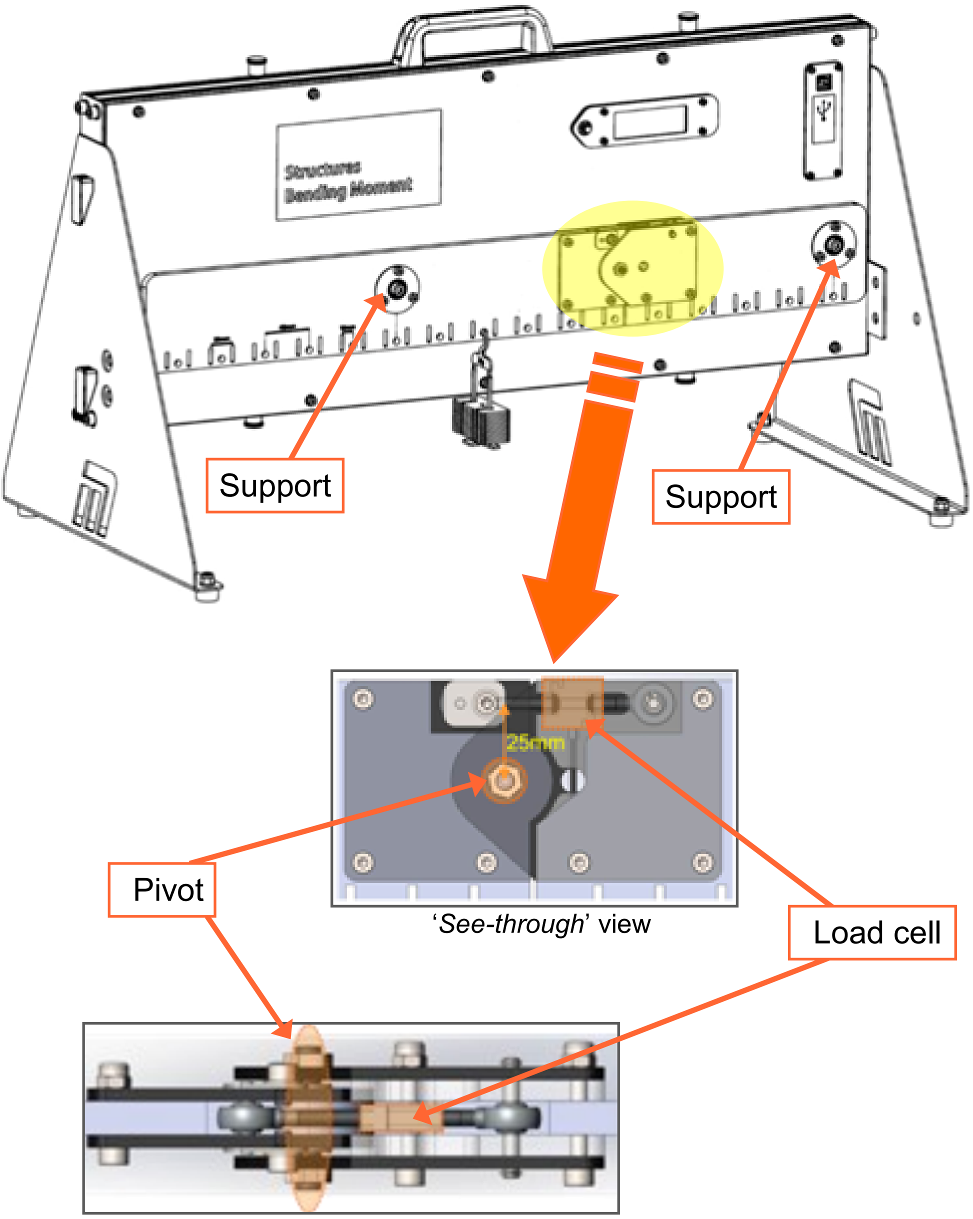
Introduction

The apparatus:

The ’Structures - Bending Moment’ apparatus allows us to investigate this bending moment. It consists of a beam, supported at two points, divided into two sections linked by a pivot.

A load cell, spanning the gap between the sections, measures the force trying to bend the beam. This force is applied at a perpendicular distance of 25mm from the pivot.
From this, the bending moment at that slice can be calculated.
The apparatus is designed to work off 5v power supply. This means that a USB cable plugged into either a computer or a plug will be sufficient. The data acquisition software only works through the computer, therefore the recommended setup is to have the USB plugged into the computer which is running the software. However, if you’d like to run the experiment without the software, a USB plug will need to be sourced for the correct local plug style.

Detail of load cell assembly:



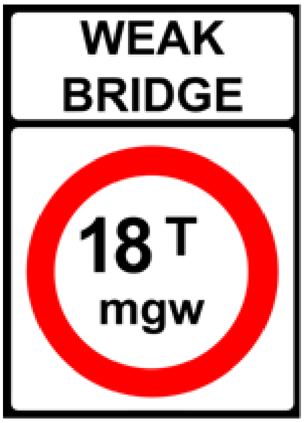
Worksheet 1

Changing the load

The design of a structure determines the loads it is capable of carrying. It can be dangerous to exceed the maximum load. For a simple beam bridge, the weakest point is usually at the centre of the span.

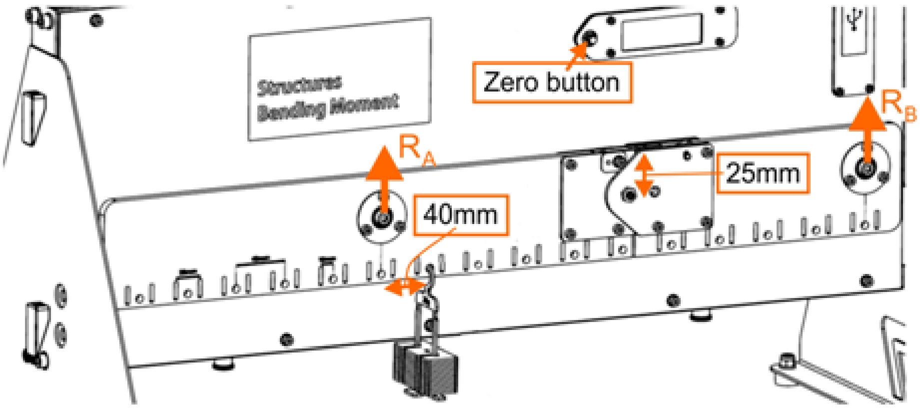
The structural engineer must be able to predict the effects of different loads on the structure.

This experiment explores the effect that changing the load on a beam has on the bending moment.



Over to you:

- Make sure that the device is level.
- Press the button on the LCD display to zero the equipment. This eliminates the weight of the beam and load cell.
- Place an empty mass hanger 40mm (one hole) away from the left-hand support, as shown in the diagram below.
- Record the load measured by the load cell, either in a spreadsheet or in the table in the Student Handout.
- Increase the mass on the hanger in 40g steps, to a maximum of 300g and record the measured load each time.



Worksheet 1

Changing the load

Over to you .....

- Calculate the measured and theoretical values of bending moment, using the formulae given in the Student Handout.
- Use the axes provided to plot graphs of measured bending moment vs suspended load and theoretical bending moment vs suspended load.

Challenge:

Use the template provided in the Student Handbook to complete the shear force diagram and bending moment diagram for this set-up when the applied load was a maximum (2.94N).

So What?

The theoretical and the experimentally measured bending moments produce very similar traces on the graphs, showing the theoretical equations for beam bending are robust and can be used to predict beam behaviour.

Worksheet 2

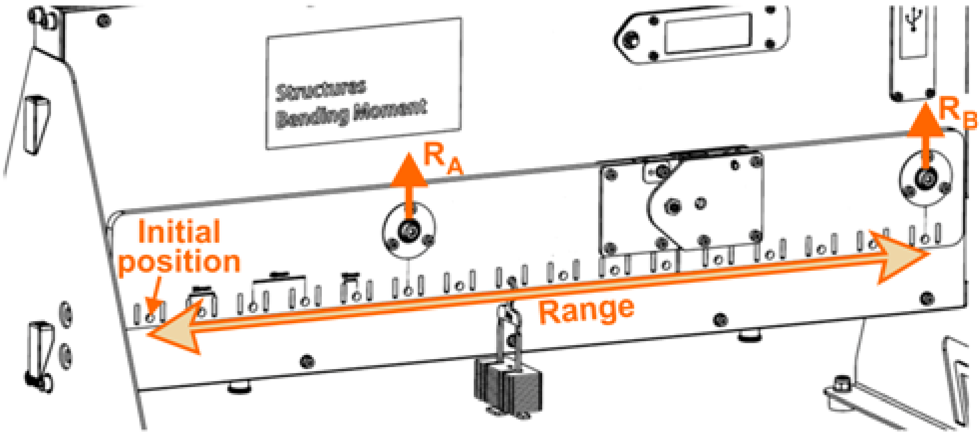
Moving the load

As a heavy load moves across a bridge, the effects of its weight on the bridge change. The engineer must predict these effects and design the structure accordingly.

This experiment explores the effect on bending moment of moving a fixed load along the beam.

Over to you:

- Make sure that the device is level.
- Press the button on the LCD display to zero the equipment and eliminate the weight of the beam and load cell.
- The experiment uses a fixed load of 300g (2.94N). Place a 300g mass hanger initially at the left-hand end of the beam, as shown in the diagram below.
- Record the load measured by the load cell either in a spreadsheet or in the table in the Student Handout.
- Move the 300g mass hanger along to the next hole on the right (i.e. 40mm from the initial position,) and record the new load cell reading.



Worksheet 2

Moving the load

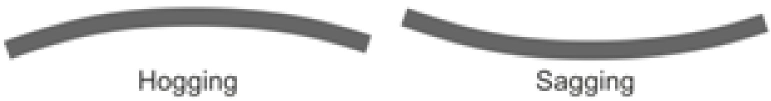
Over to you.....

- Continue in this way, moving the 300g load along the beam, hole by hole and recording the load cell readings.
- Use these readings to calculate the reaction force at support A for each position. Then calculate the measured and theoretical values of bending moment, in the same way as in the previous investigation.
- Plot graphs of measured bending moment vs distance and theoretical bending moment vs distance using the axes provided.

So What?

The bending moment changes across the length of the beam. In some locations, its polarity flips.

This results in effects known as *sagging* and *hogging*. *Hogging* describes a beam that curves upwards in the middle, and *sagging* describes a beam that curves downwards.



These effects are important in structural engineering. In ships, for example, hogging and sagging can be caused by the effect of waves when the central section of the hull is in the trough or crest between two waves, or by the distribution of the cargo within the hull.

They can cause damage to the hull in extreme cases.

In building construction, the sagging of beams can occur in beams supported at both ends. Hogging can occur in cantilever structures.

Worksheet 3

Multiple loads

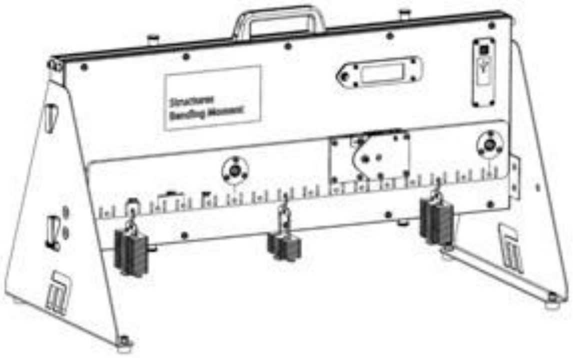
The first investigation looked at the result of changing the size of the load.

The second looked at moving that load across the structure.

In real life, structures are required to withstand both of these all the time.
This investigation explores that situation.

Outline:

Investigate the overall effect on the bending moment of placing three different loads at three different places on the beam.



Over to you:

Challenge:

- Set up three different loads in different positions on the beam.
- Record their weights, their positions and the resulting load cell reading.
- Calculate the bending moment at the slice using the measured load cell force.

- In the Student Handout, draw the free body diagram for this system.
- Apply the principle of moments to derive the reaction forces generated by the supports.
- Calculate the theoretical bending moment at the slice using one of these reaction forces.
- Check this value by calculating it again, using the other reaction force.

- Using either tables drawn in the Student Handout or a spreadsheet, record the results and your calculations.

- Repeat the same process for three different loading arrangements.

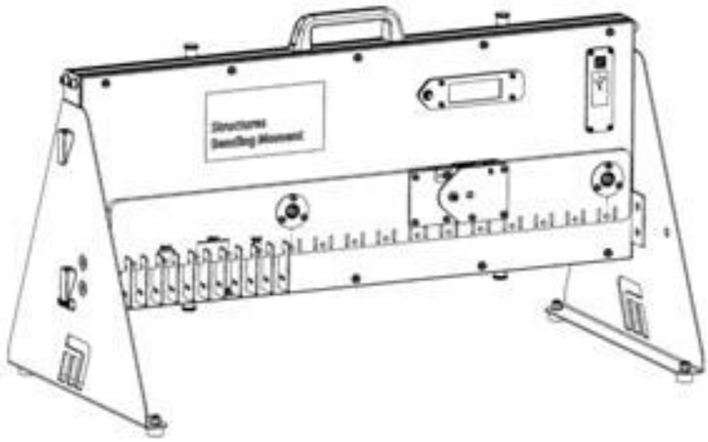
Worksheet 4

Uniformly distributed load

So far, we have considered only point loads, (also known as concentrated loads,) which act only at a specific point in the structure.
Distributed loads, on the other hand, have effects spread over a region. Most real-world loads are distributed, e.g. the weight of building materials, the force of wind or water pushing on a surface or the weight of the books on the shelf opposite.
A uniformly distributed load (UDL) has the same magnitude throughout its region of influence.
It is as if all the books on the shelf had the same size and weight.

Outline:

Investigate the effect on the bending moment
of a uniformly distributed load situated on the left-hand end of the beam.



Over to you:

Challenge:

- Set up the uniformly distributed load by placing eleven 20g masses in adjacent holes at the left-hand end of the beam, as shown in the diagram, creating a load of 2.16N.
- Record the resulting load cell reading.
- Calculate the bending moment at the slice using this measured load cell force.

- In the Student Handout, draw the free body diagram for this system.
- Use the principle of moments to derive the reaction forces generated by the supports.
- Hence, calculate the theoretical bending moment at the slice .
- Check this value by calculating it again, using the other reaction force.

- Using either tables drawn in the Student Handout or a spreadsheet, record the results and your calculations.

Student Handout

Worksheet 1 - Changing the load

Suspended mass m in g	**Load F in N**	**Load cell reading c in g**	**Load cell reading L in N**	**Measured bending moment in Nm**	**Theoretical bending moment in Nm**
20					
60					
100					
140					
180					
220					
260					
300					

Complete the table using the following formulae:

Weight = mass x gravitational field strength.

where gravitational field strength = $9.81 N.kg^{-1}$.

(Notice, all masses must be in kg.)

Hence:

F = **m** / 1000 x 9.81

L = **c** / 1000 x 9.81

Measured bending moment = **L** x 0.025

(Notice, all distances must be in m.)

Applying principle of moments to **external** forces exerting moments about support **B**:

$\mathbf{R_A} \times 0.4 = \mathbf{F} \times 0.36$

Hence calculate the value of $\mathbf{R_A}$ for each value of force **F**, and, using the formula given on p5, calculate:

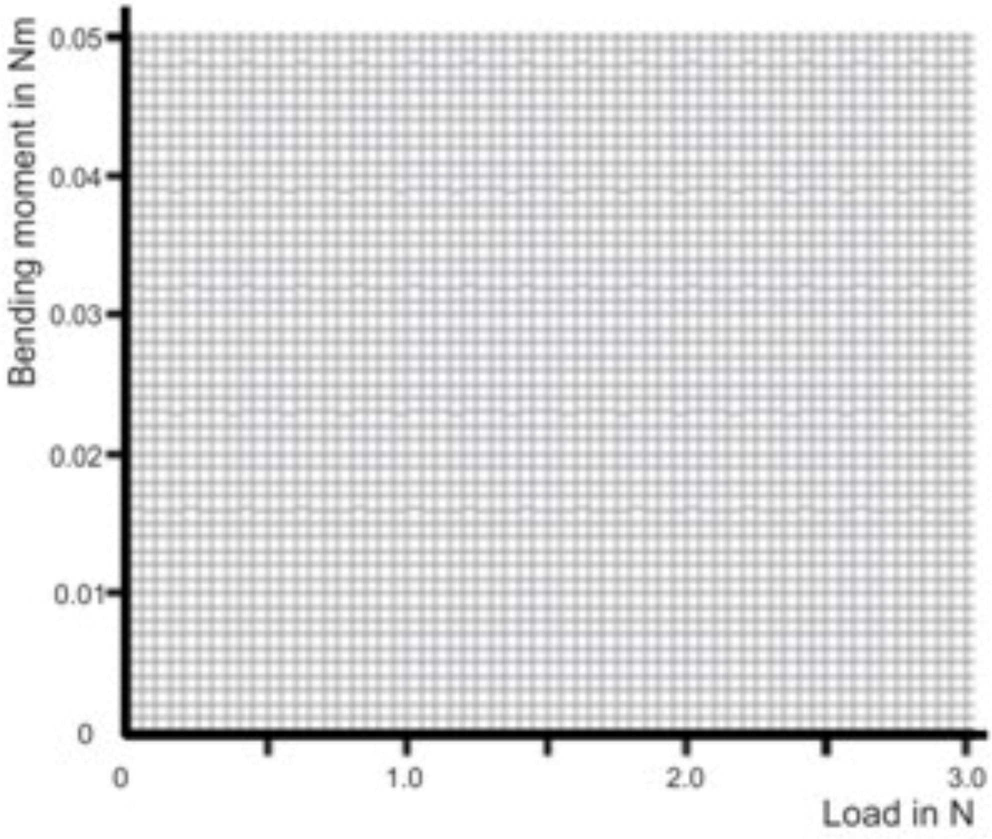
Theoretical bending moment = ($\mathbf{R_A}$ x 0.2) **-** (**F** x 0.16)

Worksheet 1 .....

On the same axes, plot graphs of **measured** bending moment vs applied load and **theoretical** bending moment vs applied load.

Graph of deflection vs load:

Show your measurements as small crosses.
Use different colours for the two traces so that you can distinguish between them easily.

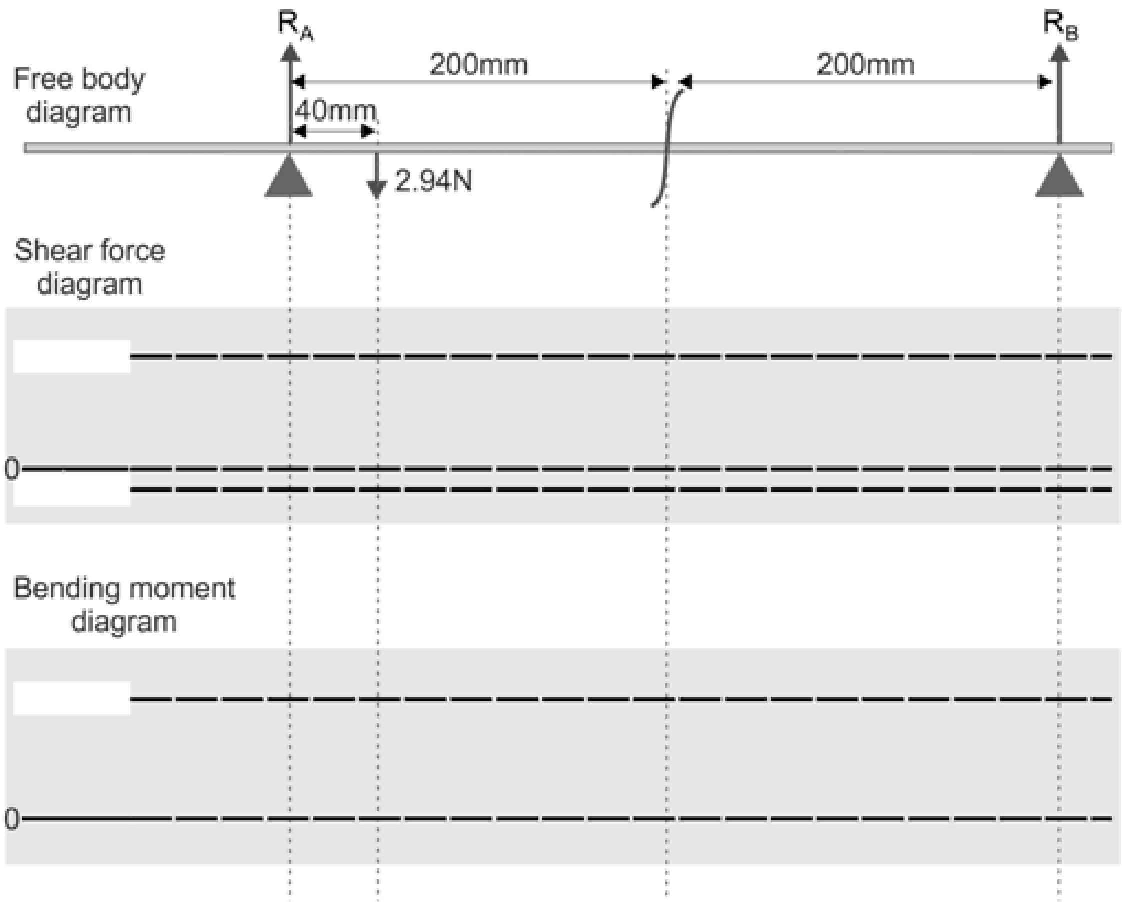


Student Handout

Worksheet 1 .....

The diagram shows the free body diagram for this arrangement when the load is 300g.

Complete the corresponding shear force diagram and bending moment diagrams.
Label significant values of shear force and bending moment.



Worksheet 2 - Moving the load

Distance x from left in m	Load cell reading c in g	Load cell reading L in N	Reaction R_A at support A in N	Measured bending moment in Nm	Theoretical bending moment in Nm
0					
0.04					
0.08					
0.12					
0.16					
0.20					
0.24					
0.28					
0.32					
0.36					
0.40					
0.44					
0.48					
0.52					
0.56					
0.60					

Complete the table using the following formulae:

Weight = mass x gravitational field strength where gravitational field strength = $9.8 N.kg^{-1}$.

(Notice, all masses must be in kg.)

The suspended 300g load weighs 0.3 x 9.81 = 2.94N

Measured bending moment = **L** x 0.025

To calculate the reaction R_A:

apply principle of moments to **external** forces exerting moments about support **B**:

$$R_A \times 0.4 = 2.94 \times (0.6 - x)$$

Then, using the formula given on p5:

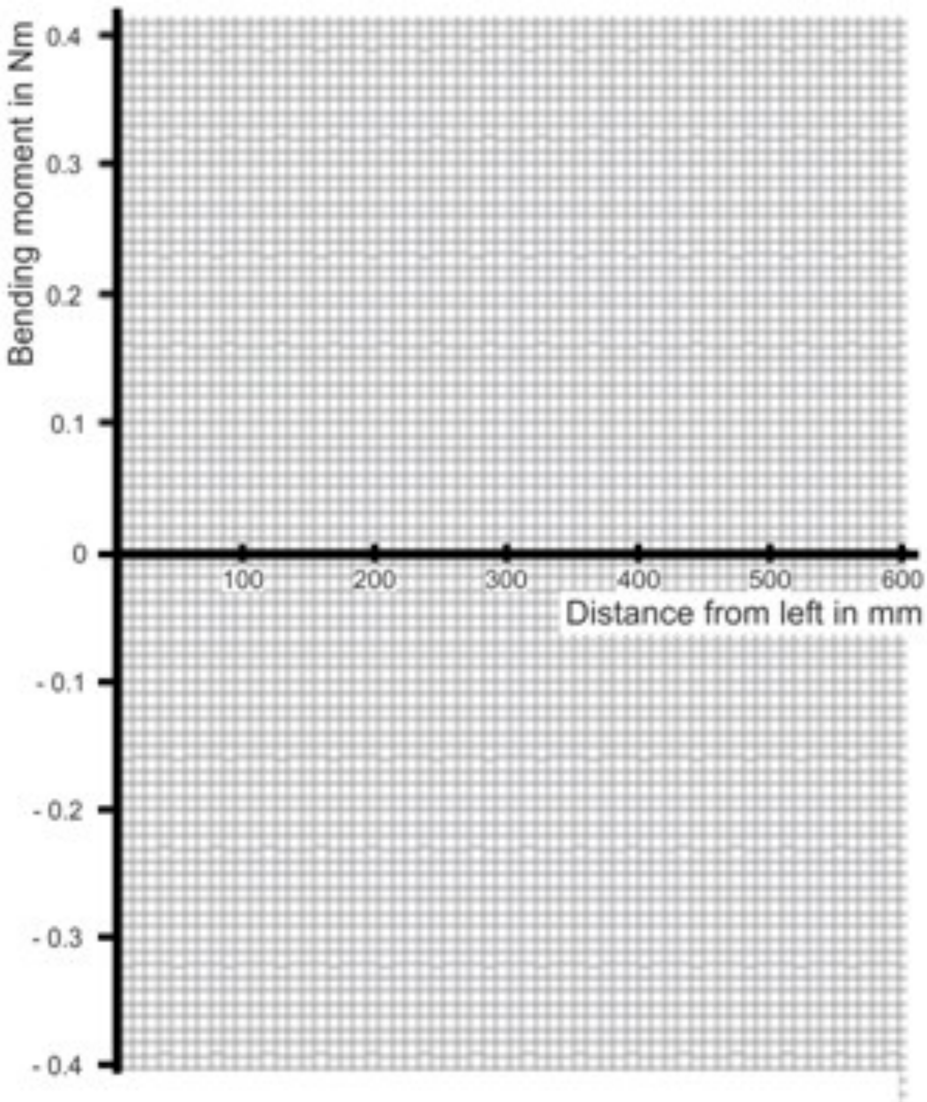
Theoretical bending moment = $(R_A \times 0.2) - [2.94 \times (0.4 - x)]$

Worksheet 2 .....

On the same axes, plot graphs of **measured** bending moment vs applied load and **theoretical** bending moment vs applied load.

Graph of deflection vs load:

Show your measurements as small crosses.
Use different colours for the two traces so that you can distinguish between them easily.



Worksheet 3 - Multiple loads

Use the following space to draw tables for your results if you wish.

Worksheet 3 - Multiple loads

- Show all your calculations and describe what they show
- Give the resulting measured and theoretical values for the bending moment at the slice.

Worksheet 4 - Uniformly distributed load

Use the following space to draw tables for your results if you wish.

Worksheet 4 - Uniformly distributed load

- Show all your calculations and describe what they show
- Give the resulting measured and theoretical values for the bending moment at the slice.

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Notes for the Instructor

Notes for the Instructor

About this course

Introduction

The ‘Structures - Bending moments’ module introduces students to the concepts of bending moment and shear force, the results of applying a load to a beam.

Using the kit, students complete a series of worksheets that focus on a number of topics found in BTEC Higher National and equivalent courses. Initially, these worksheets provide full details of the investigations. Eventually, that ‘scaffolding’ is reduced, encouraging students to demonstrate their knowledge and understanding to new situations.

Aim

The course teaches students about the relationships between applied loads and the resulting bending distortion.

Prior Knowledge

It is expected that students have followed an introductory science course, enabling them to take, record and analyse scientific observations. Some mathematical capability is required - ability to take readings from an analogue scale, ability to understand the transposition of formulae, ability to use a calculator to perform calculations and ability to plot a graph.

Using this course:

It is expected that the Worksheets and Student Handout are printed / photocopied, preferably in colour, for the students’ use.

The Student Handout is a record of measurements taken in each worksheet and questions relating to them. Students do not need a permanent copy of the worksheets but do require their own copy of the Student Handout

This format encourages self-study, with students working at a rate that suits their ability. It is for the instructor to monitor that their understanding is keeping pace with progress through the worksheets. One way to do this is to ‘sign off’ each worksheet, as the student completes it, and in the process have a brief chat to assess the student’s grasp of the ideas involved in the exercises it contains.

We realise that you as a subject area practitioner are the lead in determining how and what students learn. The worksheets are not meant to supplant this or any other supporting underpinning knowledge you choose to deliver.

For subject experts, the ’Notes for Instructors’ are provided simply to reveal the thinking behind the approach taken. For staff whose core subject knowledge is not in the field covered by the course, these notes can both illuminate and offer guidance.

Time:

It will take students between three and five hours to complete the worksheets.
A similar length of time will be needed to support the learning that takes place as a result.

Notes for the Instructor

Learning Objectives

On successful completion of this course, the student will be able to:

- describe how compressive and tensile forces in a loaded beam give rise to a bending moment;
- calculate the moment of a given force around a specified point;
- apply the principle of moments to a beam in equilibrium;
- use data about the forces acting on a beam in equilibrium and their positions to calculate the bending moment generated at a particular slice through the beam;
- describe the function of a load cell;
- ‘zero’ a load cell;
- use a load cell to obtain a measured value of bending moment;
- given data about the forces acting on a beam and their positions, draw:
    - a free-body diagram;
    - shear force diagram;
    - and bending moment diagram;

  to represent the arrangement;
- investigate how the bending moment produced by a point force changes as the point at which the force is applied moves along the beam;
- distinguish between the terms ‘hogging’ and ‘sagging’ applied to the bending of a beam;
- devise an experiment to investigate the principle of superposition of forces on a beam subjected to several simultaneous forces;
- apply the principle of superposition of forces to obtain the overall effect of multiple point forces acting simultaneously on a beam;
- devise an experiment to investigate the effect of a uniformly-distributed load on the resulting bending moment in a beam.

Notes for the Instructor

Worksheet	Notes
Introduction Timing 15 - 20 mins	Concepts involved: compressive force, tensile force, shear force, moment bending moment, couple, equilibrium, load cell The introduction seeks to show how the process causing a beam to bend can be described as the result of a bending moment and a shear force. The instructor may choose to develop further the ideas outlined here. In particular, depending on the previous experience of the students, it may be thought advantageous to explore the implications of 'equilibrium' for a body subjected to a system of forces and moments. In this way, the formulae for bending moment given on page 5 can be justified.
1 Changing the load Timing 30 - 45 mins	Concepts involved: mass, weight, gravitational field strength free body diagram, shear force diagram, bending moment diagram As this is the first time students have used this equipment, some may need reassurance with its use. Depending on their mathematical ability and experience, students may need help in understanding the argument given on page 5 which leads to a formula for the theoretical value of bending moment. The measurements are used to plot graphs that should justify the validity of the formulae used to calculate the theoretical value of bending moment. Where students are unfamiliar with the procedure for drawing free body, shear force and bending moment diagrams, the instructor will need to support them through a number of exercises before thay tackle the challenge here.
2 Moving the load Timing 30 - 45 mins	There are no new concepts. The techniques used here mirror those in the previous investigation. Then, a varying load was applied at a fixed location. This time, a constant load is moved to a variety of locations. The students need to be aware of this comparison to help them in later investigation designs. The measurements are processed in the same way as before. The significance of the sign used for the bending moment is outlined, introducing the terms 'sagging' and 'hogging'.

Notes for the Instructor

Worksheet	Notes
3 Multiple loads Timing 40 - 60 mins	There are no new concepts. The students design their own investigation into the effect of multiple loads on the bending moment. There may need to be an initial discussion on factors to consider in designing a 'fair' experiment. They could be asked to justify their approach to other groups in a class discussion. The techniques they require are those practised in the previous investigations.
4 Uniformly distributed load Timing 40 - 60 mins	Concepts involved: uniformly distributed load Once again, the students design their own investigation. This time, they investigate the effect of a uniformly distributed load on bending moment. The instructor could give an overview into types of distributed load to avoid the misapprehension that all distributed loads are uniformly distributed. Students could explore the effects of other forms of load distribution. The results of investigations could be shared via group presentations.