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Indeterminate Beam

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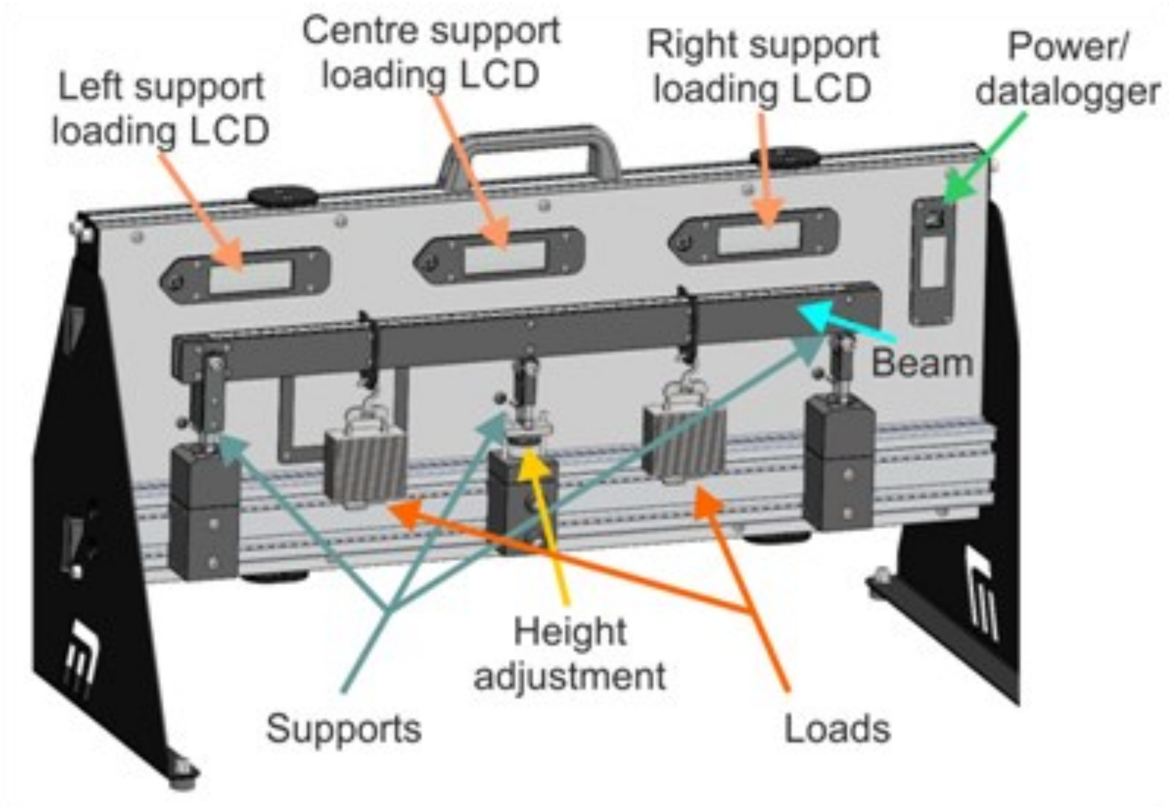
Contents

Introduction

A statically indeterminate beam is one in which unknown quantities, such as the reaction forces provided by supports, cannot be obtained using only the equations from statics - nett force = 0, nett moment = 0.

Instead we need to bring in additional equations, from consideration of the deformation of the beam, for example. The resulting mathematical treatment is complex and is not included in this course, which, instead, seeks to test its predictions.

The following diagram identifies the main components in the equipment.



The left-hand end of the beam sits on a pinned support. Elsewhere, it sits on roller supports. The reaction forces delivered by the supports are shown on LCDs. These show red and then blue backgrounds initially and then green once the reading is stable.

Worksheet 1
Simple beam

Beams are structural components that carry loads and distribute the resulting forces to the supports.
For example, in buildings, beams support upper floors and distribute their weight to columns or to the walls.
The simply-supported beam has two supports, one at each end. These restrain it and prevent any vertical or horizontal movement at the ends .

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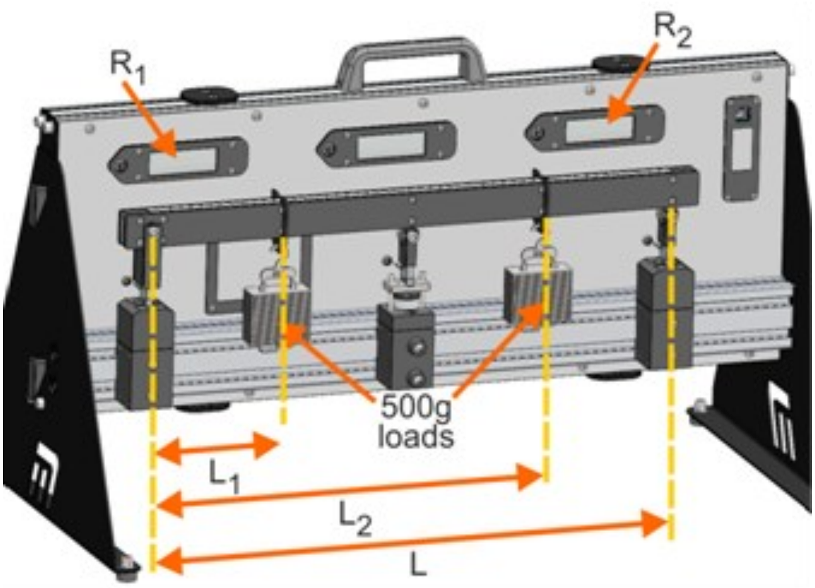

- Set up the hardware as shown in the diagram above with empty mass hangers on the beam.
- On the middle support, rotate the height adjustment wheel clockwise, as seen from above, to lower it as far as possible. It is not used in this investigation.
- Press the 'zero' button on each LCD display to initialise the readings.
- Place two 500g loads at distances L_1 =5cm and L_2 =30cm from the left-hand support.
- Record the distances and the resulting reactions R_1 and R_2, shown on the LCD displays, in Table 1 of the Student Handout.
- Repeat the same procedure with the loads in three other positions, recording the resulting reactions R_1 and R_2 and the distances L_1 and L_2 in the Student Handout for each, as before.

Worksheet 1
Simple beam

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So what:

Since the beam is in equilibrium:

- the supports provide upward forces that balance the weights on the beam;
- the clockwise moments of the forces are balanced by the anticlockwise moments, calculated around any point.

In other words:

- the reaction forces R_1 and R_2 always add up to the total load on the beam
- Taking moments about the left-hand support:

$$L_1 \times 500 + L_2 \times 500 = R_2 \times L$$

where L is the length of the beam measured between the supports

- The reaction force R_2 has no moment (turning effect) about the left-hand support.

Complete Table 2 in the Student Handout and comment on how well your results support this equation.

Challenges:

- Draw a free-body diagram for this arrangement.
- Devise an experiment based on this arrangement to measure the mass of the unloaded beam.

Worksheet 2

Three supports - single point load

More complex structures involve more complicated load distribution. The image shows the 'skybridge' walkway between the Petronas towers in Kuala Lumpur, the capital of Malaysia.
In addition to the supports at the ends, this uses a third support, half-way along its length.

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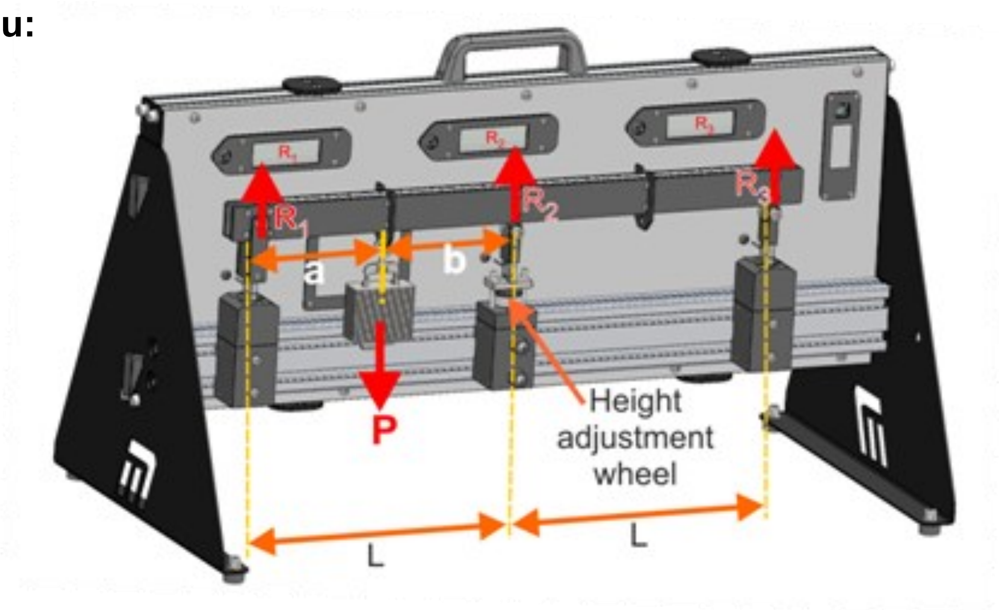


- Set up the hardware as shown in the diagram above, with two mass hangers on the beam but without the load attached.

  In this arrangement, the supports are placed symmetrically to provide equal spans.
- From theory, the reactions at the three supports are given by:

$$R_1 = \frac{P.b.}{4.L^3}(4L^2 - a.(L + a)) \qquad R_2 = \frac{P.a.}{4.L^3}(2L^2 + b.(L + a)) \qquad R_3 = -\frac{P.a.b.}{4.L^3}(L + a)$$

  Use these to calculate the three reaction forces, R_1, R_2 and R_3 when:

  P = 500g   L = 250mm   a = 125mm   b = 125mm
- Record your results in Table 1 in the Student Handout.

Over to you.....

- On the middle support, rotate the height adjustment wheel clockwise, as seen from above, to lower it as far as possible.
- Zero all the readings.
- Position a mass hanger 125mm from the left-hand support and hang a load of 500g from it.
- Rotate the height adjustment wheel anticlockwise to raise the middle support until the centre LCD shows a reading as close as possible to the theoretical value you calculated earlier.
- In Table 2 in the Student Handout, record the values shown on the LCDs for the three reaction forces.
- Repeat the procedure for the other loads listed in the table.

  For each:
    - recalculate the predicted values for the reaction forces;
    - reset the centre support to its lowest position;
    - zero the LCDs;
    - raise the centre support until the centre LCD reading matches, as closely as possible, the theoretical value for the reaction force;
    - read and record all reaction forces in Table 2.
- Plot graphs showing how each of the three reaction forces changes as the load increases.

Challenges:

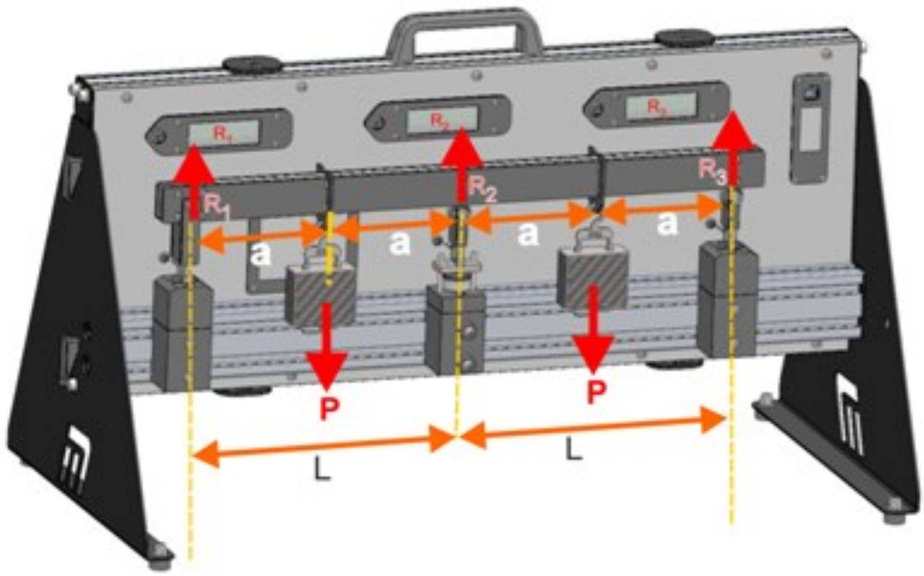
- In the Student Handout, explain why the value of R_3 is negative.
- Draw a free-body diagram for this arrangement.

Worksheet 3
Three supports - two point loads

Structural components may be subject to forces from different types of load:

- point loads, where forces act over a very small area;
- distributed loads, where forces act over an extended area;
- couple loads, consisting of two equal forces acting in opposite directions.

Over to you:



- Set up the hardware as shown in the diagram above, with two mass hangers in place on the beam but without loads attached.

  As before, the supports are placed symmetrically to provide equal spans.
- From theory, the reactions at the three supports are given by:

$$R_1 = R_3 = \frac{5.P}{16} \qquad R_2 = \frac{11.P}{8}$$

  Use these to calculate the three reaction forces, R_1, R_2 and R_3 when P = 500g.
- Record your results in Table 1 in the Student Handout.

Worksheet 3
Three supports - two point loads

Over to you.....

- The procedure is similar to that followed in worksheet 2:
  - on the middle support, rotate the height adjustment wheel clockwise, as seen from above, to lower it as far as possible;
  - zero all the readings;
  - position one mass hanger 125mm from the left-hand support and the other 125mm away from the right-hand support, i.e. at the centres of the spans;
  - hang loads of 500g from each;
  - rotate the height adjustment wheel anticlockwise to raise the middle support until the centre LCD shows a reading as close as possible to the theoretical value calculated earlier;
  - in Table 2 in the Student Handout, record the values shown on the LCDs for the three reaction forces;
  - repeat the procedure for the other loads listed in Table 1.
- Again, for each:
  - recalculate the predicted values for the reaction forces;
  - reset the centre support to its lowest position;
  - zero the LCDs;
  - raise the centre support until the centre LCD matches, as closely as possible, the theoretical value for the reaction force;
  - read and record all reaction forces in Table 2.
- Plot graphs showing how each of the three reaction forces changes as the load increases.

Challenges:

- Draw a free-body diagram for this arrangement.
- Explain why the value of R_3 is no longer negative.

Worksheet 4

Unequal spans - single point load

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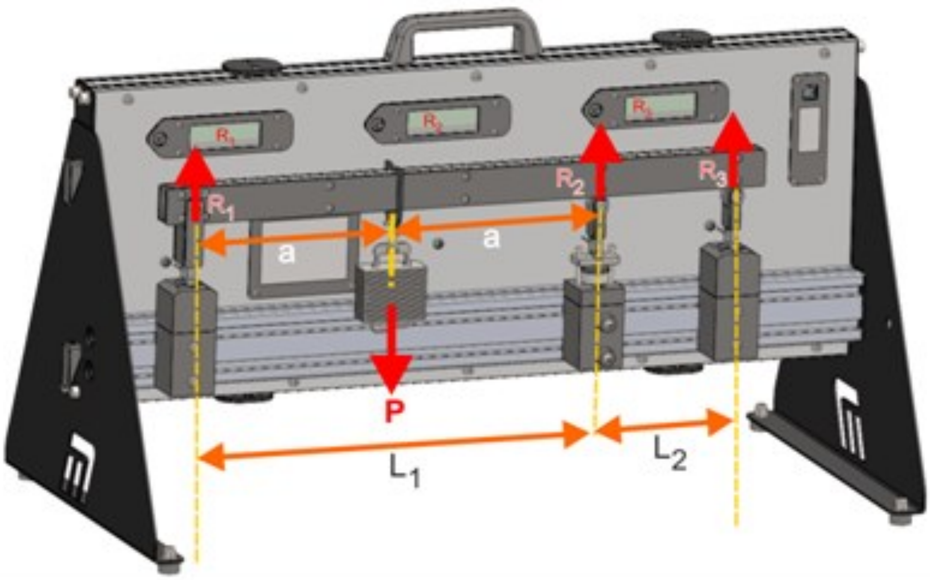
For reasons of geography, soil structure or location, structures such as bridges are often designed having a number of spans of different lengths.
Some of these are exposed to frequent excitation by road or rail traffic. In this case, uneven spans may improve their mechanical properties.

Over to you:



- Set up the hardware as shown above, with two empty mass hangers in place.
  Here, the supports are no longer placed symmetrically, giving unequal spans.
- The theoretical analysis of this arrangement uses M_1, the bending moment at the centre support, to calculate the reactions at the other two supports:

$$M_1 = -\frac{3}{16}\left(\frac{P \cdot L_1^2}{L_1 + L_2}\right) \qquad R_1 = \frac{M_1}{L_1} + \frac{P}{2} \qquad R_2 = P - R_1 - R_3 \qquad R_3 = \frac{M_1}{L_2}$$

  Use these to calculate the three reaction forces, R_1, R_2 and R_3 when P = 500g, L_1 = 375mm and L_2 = 125mm.
- Record your results in Table 1 in the Student Handout.

Worksheet 4
Unequal spans - single point load

Over to you.....

- Loosen the fixings on the middle support and slide it to a position 375mm from the left-hand support. Tighten the fixings to keep it in that position.
- The procedure is then similar to that followed in earlier worksheets:
  - on the middle support, rotate the height adjustment wheel clockwise, as seen from above, to lower it as far as possible;
  - zero all the readings;
  - position a mass hanger at a distance of 187.5mm (1/2 x $\mathbf{L_1}$) from the left-hand support;
  - hang a load of 500g from it;
  - rotate the height adjustment wheel anticlockwise to raise the middle support until the centre LCD shows a reading as close as possible to the theoretical value calculated earlier;
  - in Table 2 in the Student Handout, record the values shown on the LCDs for the three reaction forces;
  - repeat the procedure for the other loads listed in Table 1.
- Again, for each value of load:
  - calculate the predicted values for each of the reaction forces;
  - reset the centre support to its lowest position;
  - zero the LCDs;
  - hang the load from the mass hanger and check its position on the beam
  - raise the centre support until the centre LCD matches, as closely as possible, the theoretical value for the reaction force;
  - read and record all reaction forces in Table 2.
- Plot graphs showing how each of the three reaction forces changes as the load increases.

Challenge:

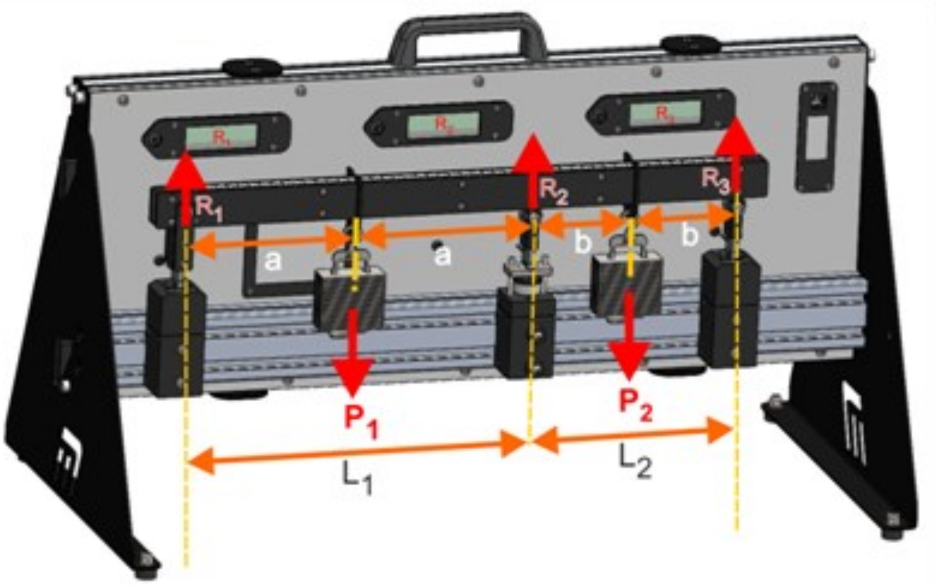
- Repeat the experiment for different span lengths.

Worksheet 5

Unequal spans - two point loads

In real structures, the lengths of the spans may vary and there can be a variety of loads applied to them. As a step in that direction, this setup looks at just two unequal spans supporting two point loads.

Over to you:



- Set up the hardware with two mass hangers in place but no loads attached. Once again, the supports are not placed symmetrically, giving unequal spans.
- The theoretical analysis of this arrangement again uses M_1, the bending moment at the centre support, to calculate the reactions at the other two supports:

$$M_1 = -\frac{3}{16}\left(\frac{P_1 . L_1^2 + P_2 . L_2^2}{L_1 + L_2}\right) \qquad R_1 = \frac{M_1}{L_1} + \frac{P_1}{2} \qquad R_2 = P_1 + P_2 - R_1 - R_3 \qquad R_3 = \frac{M_1}{L_2} + \frac{P_2}{2}$$

  Use these to calculate the three reaction forces, R_1, R_2 and R_3 when P_1 and P_2 = 500g, L_1 = 300mm and L_2 = 200mm.
- Record your results in Table 1 in the Student Handout.

Worksheet 5
Unequal spans - two point loads

Over to you.....

- Move the middle support to a position 300mm from the left-hand support and clamp it there.
- The procedure is then similar to that followed in earlier worksheets:
  - on the middle support, rotate the height adjustment wheel clockwise, as seen from above, to lower it as far as possible;
  - zero all the readings;
  - position a mass hanger at a distance of 150mm (1/2 x $\mathbf{L_1}$) from the left-hand support and hang a load of 500g from it;
  - position the second mass hanger at a distance of 100mm (1/2 x $\mathbf{L_2}$) from the right-hand support and hang a load of 500g from it;
  - rotate the height adjustment wheel anticlockwise to raise the middle support until the centre LCD shows a reading as close as possible to the theoretical value calculated earlier;
  - in Table 2 in the Student Handout, record the values shown on the LCDs for the three reaction forces;
  - repeat the procedure for the other loads listed in Table 1.
- Again, for each value of load:
  - calculate the predicted values for each of the reaction forces;
  - reset the centre support to its lowest position;
  - zero the LCDs;
  - hang the loads from the mass hangers and check their positions on the beam
  - raise the centre support until the centre LCD matches, as closely as possible, the theoretical value for the reaction force;
  - read and record all reaction forces in Table 2.
- Plot graphs showing how each of the three reaction forces changes as the load increases.

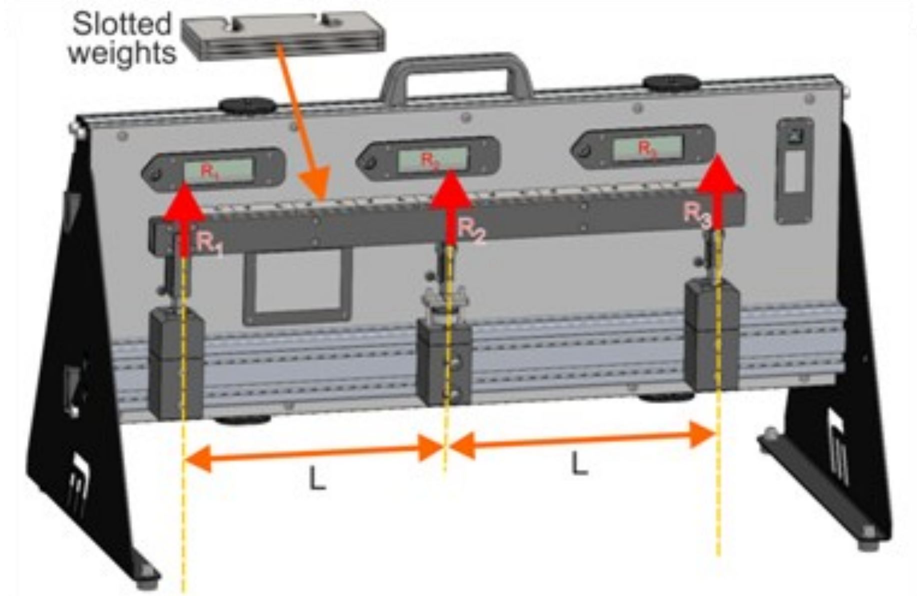
Challenge:

- As for the previous worksheet, repeat the experiment for different span lengths.

Worksheet 6
Equal spans - distributed load

A distributed load is one that is spread out along the length of a beam rather than being concentrated at a single point. In reality, most loads are distributed.
Examples include forces due to wind or water, pushing on a surface, the weight of snow on a building roof and the weight of building materials themselves,.
In a uniformly distributed load, (UDL), the force per unit length on the structure is the same at every point.

Over to you:



- Set up the hardware as shown above:
  - the centre support is moved back to its original position to create two equal spans;
  - the two mass hangers are removed.
- Theoretical analysis of this arrangement uses the quantity **w**, the load per unit length for the distributed load, to calculate the reactions at the supports.
  In this case, **w** = (9 x 60) / 500 = 1.08 g.mm^{-1}.
  Using this, the reactions at the three supports are given by:

$$R_1 = R_3 = \frac{3}{8} \cdot (\mathbf{w} \cdot L) \quad R_2 = \frac{10}{8} \cdot (\mathbf{w} \cdot L)$$

  Use these to calculate the three reaction forces, R_1, R_2 and R_3, given that L = 250mm.
- Record your results in Table 1 in the Student Handout.

Over to you.....

- The procedure is similar to that followed in earlier worksheets:
    - on the middle support, rotate the height adjustment wheel clockwise, as seen from above, to lower it as far as possible;
    - zero all the readings;
    - place nine stacks of three 20g masses centrally on the beam to create a distributed load.
    - rotate the height adjustment wheel anticlockwise to raise the middle support until the centre LCD shows a reading as close as possible to the theoretical value calculated earlier;
    - in Table 2 in the Student Handout, record the values shown on the LCDs for the three reaction forces.

Challenge:

Investigate the effect on the reaction forces of having this uniform load distributed only across one of the spans.

Begin by predicting which reaction forces will increase and which will decrease. Record your predictions in the Student Handout.

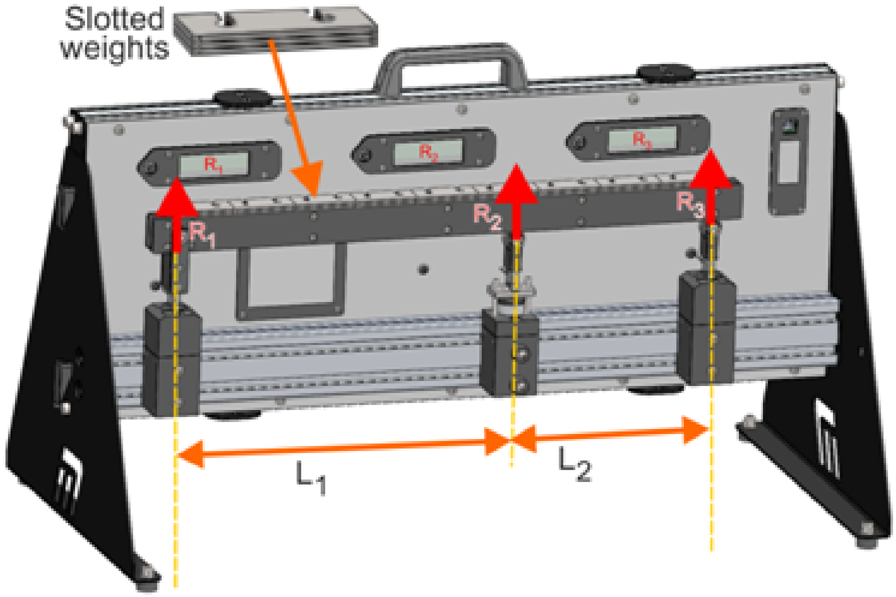
Then carry out an experiment to test your predictions. Give the results of this in the Student Handout.

Worksheet 7

Unequal spans - distributed load

In practical structures, the weight of the components creates a distributed load across the spans.
A balance must be struck between using lightweight materials to keep that weight down and ensuring that there is sufficient strength to support the structure.

Over to you:



- Set up the hardware as shown above.
  The centre support is displaced to one side to create unequal spans.
- Theoretical analysis of this arrangement uses the quantity **w**, the load per unit length, for the distributed load, to calculate the reactions at the supports.
  In this case, **w** = (9 x 60) / 500 = 1.08 g.mm^{-1}.
  Using this, the reactions at the three supports are given by:

$$M_1 = -\frac{\mathbf{w} \cdot L_2^2 + \mathbf{w} \cdot L_1^2}{8(L_1 + L_2)}$$

$$R_1 = \frac{M_1}{L_1} + \frac{\mathbf{w} \cdot L_1}{2} \qquad R_2 = \mathbf{w} \cdot L_1 + \mathbf{w} \cdot L_2 - R_1 - R_3 \qquad R_3 = \frac{M_1}{L_2} + \frac{\mathbf{w} \cdot L_2}{2}$$

  Use these to calculate the three reaction forces, R_1, R_2 and R_3, given that L_1 = 300mm and L_2 = 200mm.
- Record your results in Table 1 in the Student Handout.

Worksheet 7
Unequal spans - distributed load

Over to you.....

- The procedure is similar to that followed in the last worksheet:
    - on the middle support, rotate the height adjustment wheel clockwise, as seen from above, to lower it as far as possible;
    - zero all the readings;
    - place nine stacks of three 20g masses centrally on the beam to create a distributed load.
    - rotate the height adjustment wheel anticlockwise to raise the middle support until the centre LCD shows a reading as close as possible to the theoretical value calculated earlier;
    - in Table 2 in the Student Handout, record the values shown on the LCDs for the three reaction forces.

Challenge:

Investigate the effect on the reaction forces of having a non-uniform load, distributed across two unequal spans.

Once again, try to predict which reaction forces will increase and which will decrease. Record your predictions in the Student Handout.

Then carry out an experiment to test your predictions. Give the results of this in the Student Handout.

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Student Handout

Worksheet 1 - Simple beam

Table 1

Two loads each of mass 500g					
Setup	**Distance L_1 from load 1 to left-hand support in cm**	**Distance L_2 from load 2 to left-hand support in cm**	**Reaction R_1 in gf**	**Reaction R_2 in gf**	**R_1+R_2 in gf**
1	5	30			
2					
3					
4					

Table 2

Two loads each of mass 500g		
Setup	**L_1 x 500 + L_2 x 500**	**R_2 x L**
1		
2		
3		
4		

Comment on the significance of the results in Table 2.

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Challenges:

Free-body diagram for this arrangement:

Estimate of mass of the unloaded beam (from experiment):

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Worksheet 2 - Three supports - single point load

Using these predictions from theory:

$$R_1 = \frac{P.b}{4.L^3}.(4L^2 - a.(L + a)) \qquad R_2 = \frac{P.a}{4.L^3}.(2L^2 + b.(L + a)) \qquad R_3 = -\frac{P.a.b}{4.L^3}.(L + a)$$

when P = 500g L = 250mm a = 125mm b = 125mm

Table 1

Theoretical predictions			
Load P in g	Reaction R_1 in gf	Reaction R_2 in gf	Reaction R_3 in gf
500			
400			
300			
200			
100			

Table 2

Measured values			
Load P in g	Reaction R_1 in gf	Reaction R_2 in gf	Reaction R_3 in gf
500			
400			
300			
200			
100			

Comment on the comparison between the theoretical and measured values.

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Challenges:

Free-body diagram for this arrangement:

Explain why the value of R_3 is negative:

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Worksheet 3 - Three supports - two point loads

Using these predictions from theory: $R_1 = R_3 = \frac{5.P}{16}$ $R_2 = \frac{11.P}{8}$

when both loads (P) = 500g

Theoretical predictions			
Load P in g	Reaction R_1 in gf	Reaction R_2 in gf	Reaction R_3 in gf
500			
400			
300			
200			
100			

Table 1

Measured values			
Load P in g	Reaction R_1 in gf	Reaction R_2 in gf	Reaction R_3 in gf
500			
400			
300			
200			
100			

Table 2

Again, comment on the comparison between the theoretical and measured values.

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Challenges:

Free-body diagram for this arrangement:

Explain why the value of R_3 is no longer negative:

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Worksheet 4 - Unequal spans - one point load

Using these predictions from theory:

$$M_1 = -\frac{3}{16}\left(\frac{P \cdot L_1^2}{L_1 + L_2}\right) \qquad R_1 = \frac{M_1}{L_1} + \frac{P}{2} \qquad R_2 = P - R_1 - R_3 \qquad R_3 = \frac{M_1}{L_2}$$

when P = 500g, L_1 = 375mm and L_2 = 125mm:

Table 1

Theoretical predictions			
Load P in g	**Reaction R_1 in gf**	**Reaction R_2 in gf**	**Reaction R_3 in gf**
500			
400			
300			
200			
100			

Table 2

Measured values			
Load P in g	**Reaction R_1 in gf**	**Reaction R_2 in gf**	**Reaction R_3 in gf**
500			
400			
300			
200			
100			

Again, comment on the comparison between the theoretical and measured values.

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Worksheet 5 - Unequal spans - two point loads

Using these predictions from theory:

$$M_1 = -\frac{3}{16}\left(\frac{P_1 . L_1^2 + P_2 . L_2^2}{L_1 + L_2}\right) \quad R_1 = \frac{M_1}{L_1} + \frac{P_1}{2} \quad R_2 = P_1 + P_2 - R_1 - R_3 \quad R_3 = \frac{M_1}{L_2} + \frac{P_2}{2}$$

when P = 500g, L_1 = 300mm and L_2 = 200mm:

Table 1

Theoretical predictions			
Load P in g	**Reaction R_1 in gf**	**Reaction R_2 in gf**	**Reaction R_3 in gf**
500			
400			
300			
200			
100			

Table 2

Measured values			
Load P in g	**Reaction R_1 in gf**	**Reaction R_2 in gf**	**Reaction R_3 in gf**
500			
400			
300			
200			
100			

Again, comment on the comparison between the theoretical and measured values.

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Worksheet 6 - Equal spans - distributed load

Using these predictions from theory:

$$R_1 = R_3 = \frac{3}{8} \cdot (\mathbf{w} \cdot L) \quad R_2 = \frac{10}{8} \cdot (\mathbf{w} \cdot L)$$

when **w** = 1.08g.mm^{-1} and L = 250mm:

Table 1

Theoretical predictions		
Reaction R_1 in gf	Reaction R_2 in gf	Reaction R_3 in gf

Table 2

Measured values		
Reaction R_1 in gf	Reaction R_2 in gf	Reaction R_3 in gf

Challenge:

Predict what will happen when the same UDL is applied across only one of the spans:

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Experimental results:

Measured values		
Reaction R_1 in gf	Reaction R_2 in gf	Reaction R_3 in gf

Worksheet 7 - Unequal spans - distributed load

Using these predictions from theory:

$$M_1 = \frac{-\mathbf{w} \cdot L_2^2 + \mathbf{w} \cdot L_1^2}{8(L_1 + L_2)}$$

$$R_1 = \frac{M_1}{L_1} + \frac{\mathbf{w} \cdot L_1}{2} \qquad R_2 = \mathbf{w} \cdot L_1 + \mathbf{w} \cdot L_2 - R_1 - R_3 \qquad R_3 = \frac{M_1}{L_2} + \frac{\mathbf{w} \cdot L_2}{2}$$

when **w** = 1.08g.mm^{-1} L_1 = 300mm and L_2 = 200mm:

Table 1

Theoretical predictions		
Reaction R_1 in gf	Reaction R_2 in gf	Reaction R_3 in gf

Table 2

Measured values		
Reaction R_1 in gf	Reaction R_2 in gf	Reaction R_3 in gf

Challenge:

Predict what will happen when a non-uniform distributed load is applied across a beam divided into two unequal spans:

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Sketch of non-uniform load:

Experimental results:

Measured values		
Reaction R_1 in gf	Reaction R_2 in gf	Reaction R_3 in gf