MATRIX | STRUCTURES

Two & Three Pinned Arch

MATRIX

CP4922

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Contents

MATRIX STRUCTURES

Two and Three Pinned Arch

Introduction

Arches have been used to support bridges for centuries. The Jade Belt bridge, shown above, dates back to 18th-centuary.

They offer a number of advantages for bridge building. For example, for the same amount of material, it can have larger span and carry more weight than a flat beam bridge.

Arch bridges are built either as a fixed arch, a two-pinned (hinged) arch or a three-pinned (hinged) arch.

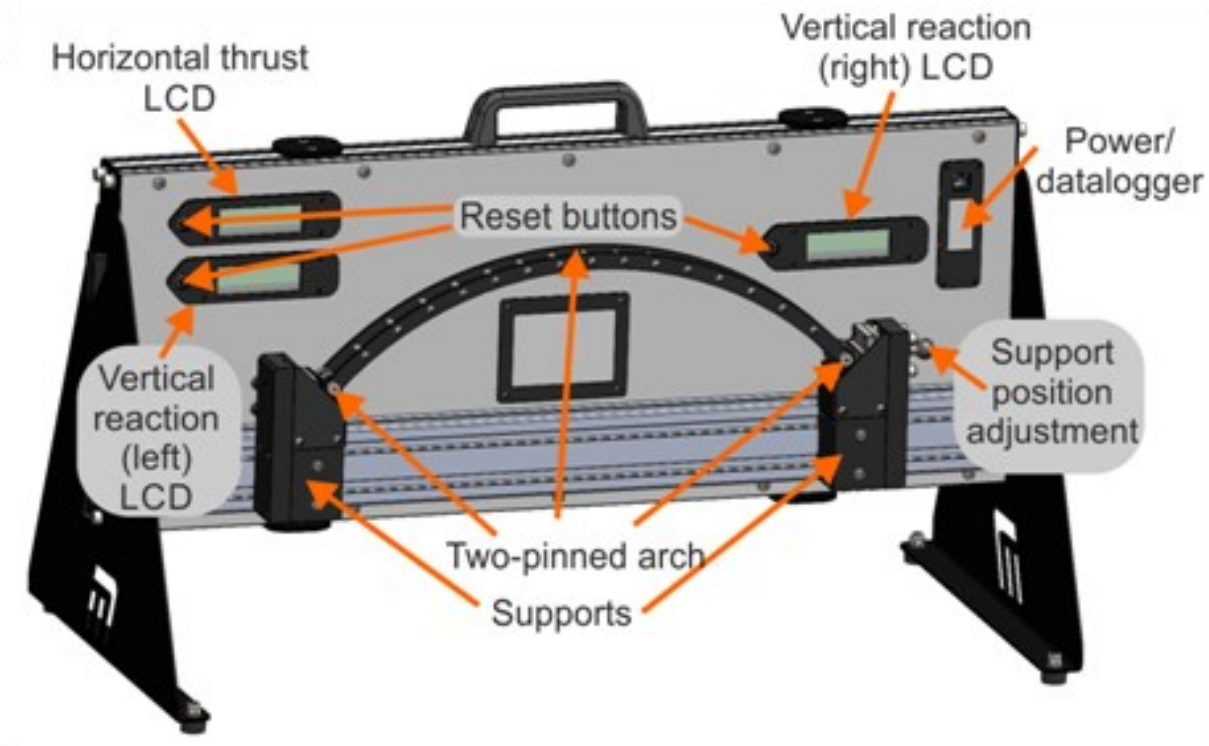
- The fixed arch is most often used for relatively short spans, usually built in concrete.
- The two-pinned arch is used to bridge longer spans and allows the structure to move more freely.
- The three-pinned arch usually has an additional pivot positioned at its centre, which allows it even greater freedom of movement to compensate for the effects of temperature change, for example.

Introduction

This module examines factors affecting the design of arches. These are found supporting a wide range of objects from bridge decks to openings in walls and to park benches.
Their main structural advantage over a straight beam, is that they can carry much larger loads.

The kit allows students to apply a variety of loads to a two-pinned circular arch and a three-pinned circular arch, in order to explore the resulting forces in the supports.
For example, they can move a concentrated load across the span of the arch or apply types of UDL and measure the resulting forces, allowing them to compare theoretical and experimental values. Although it may not appear so, moving the load from one rung to the next increases the horizontal displacement of the load by 25mm each time.

The following diagram identifies the main components in the equipment.



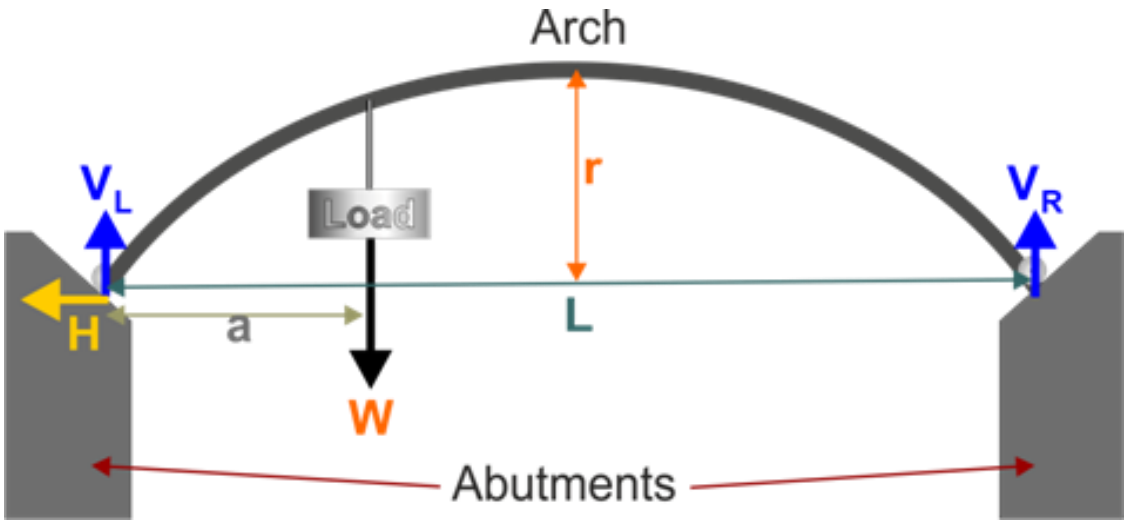
Two load cells, connected to LCDs, measure the vertical reaction force, $\mathbf{V_L}$ and the horizontal thrust, **H**, produced at the left-hand end of the kit. At the right-hand end, a similar arrangement displays the vertical reaction force, $\mathbf{V_R}$. (As the arch is in equilibrium, there is an equal but opposite horizontal thrust applied to the arch at this end.)
The horizontal position of the right-hand support can be altered using the adjustment screw located in the end support.

Introduction

Taking Measurements:

Internally, the load cells are controlled by software that includes a sensitivity function. If the sensitivity is set too low, then readings will fluctuate due to electrical noise in the system. If it is set too high, then the load cells become insensitive to changes in the loading.

The setting used for sensitivity, then, is a compromise. Increasing the load by a small amount may not produce a change in the LCD reading if it is smaller than the sensitivity 'step'. To overcome this, press gently on the top of the arch until the LCD reading changes. When you release this pressure, the reading will show the new true value of load.



Key to symbols:

- **a** = horizontal distance from left-hand pin to suspension point in mm
- **H** = horizontal thrust measured at left-hand support in g
- **V_L** = vertical reaction force measured at left-hand support in g
- **V_R** = vertical reaction force measured at right-hand support in g
- **W** = load applied
- **L** = span of arch = 400mm
- **r** = height of arch = 85mm

Worksheet 1
Effect of position of point load

MATRIX STRUCTURES

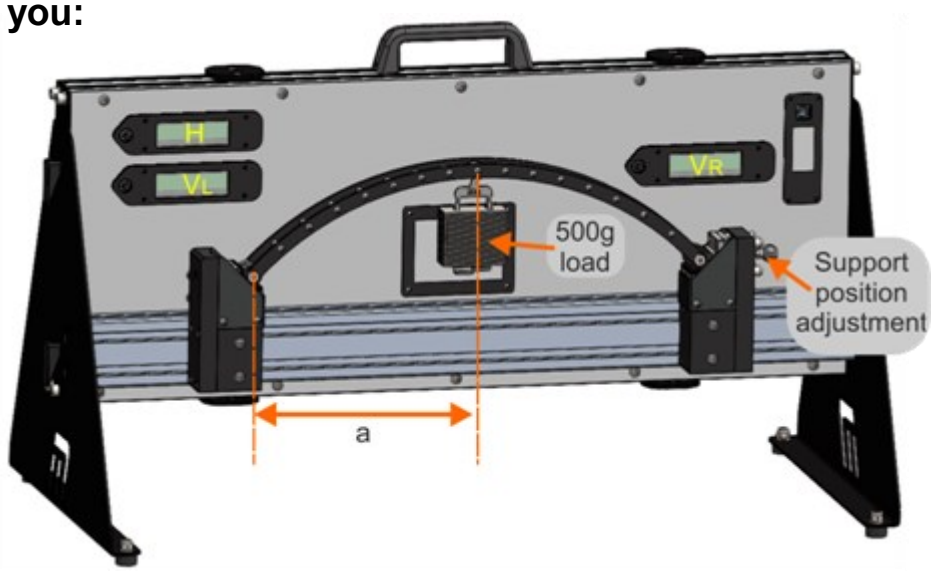
Two and Three Pinned Arch

In bridge design, the forces generated by a vehicle change as it moves across.

The design must ensure that the members and supports used in the bridge can cope with this.

A. Two-pinned arch

Over to you:



- Set up the equipment as shown in the diagram, with the arch touching the left-hand support.
- Turn the support position adjustment to move the right-hand support inwards until it just touches the other end of the arch. (At this point, the reading on one of the LCD's will change.)
- Press the reset button on each LCD display to initialise the readings.
- Suspend a 500g load from the second rung of the arch, (distance **a** = 50mm.)
- Remember to touch the crown of the arch gently to ensure that the load cells register the load properly.
- Observe the values of the readings $\mathbf{V_L}$, $\mathbf{V_R}$ and **H**.
- Record these measurements in table 1 in the Student Handout.
- Remove the load and reset the LCD displays.
- Now suspend the 500g load from the third rung of the arch, (distance **a** = 75mm.)
- Repeat the procedure.
- Repeat this procedure for the other distances given in table 1.

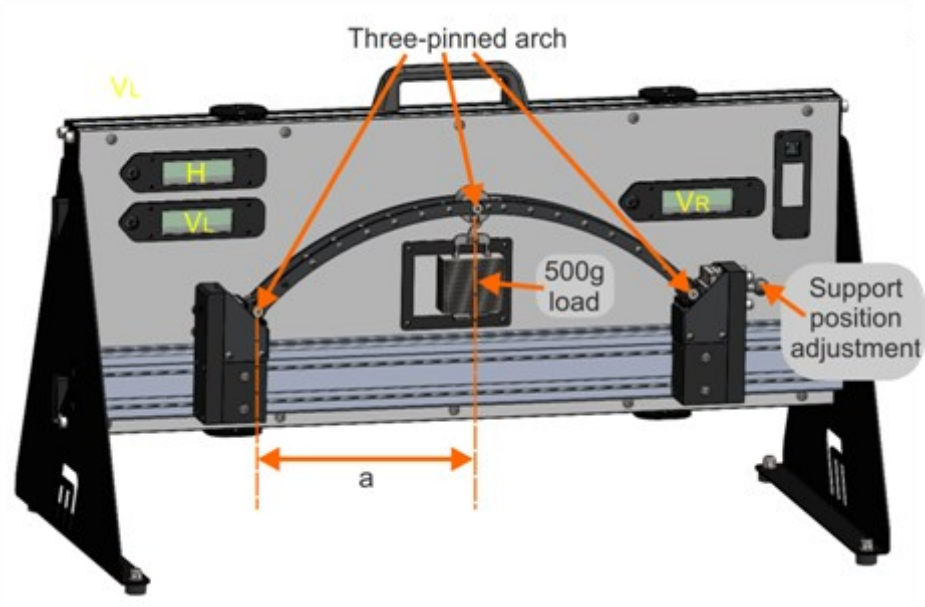
Worksheet 1
Effect of position of point load

So what:

- Using the axes provided in the Student Handout, use your results to plot graphs of:
  - vertical reaction, $\mathbf{V_L}$, at the left-hand end of the arch, vs distance **a**;
  - vertical reaction, $\mathbf{V_R}$, at the right-hand end of the arch, vs distance **a**;
  - horizontal thrust **H** vs distance **a**.
- Draw smooth curves through your experimental points.
- Use the formulae given in the Student Handout to calculate theoretical values of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** for the values of distance **a** used in the experiment and add them to the results table.
- Plot these values on the graph and draw straight lines through them.

B. Three-pinned arch

Over to you:



- Replace the two-pinned arch with the three-pinned arch, as shown in the diagram, keeping the right-hand support in the same position.
- Press the reset button on each LCD display to initialise the readings.
- Repeat the same procedure as in part A, moving the load to the positions shown in table 2.
- Observe the values of the readings $\mathbf{V_L}$, $\mathbf{V_R}$ and **H**.
- Record these measurements in table 2 in the Student Handout.

Worksheet 1
Effect of position of point load

So what:

- As before:
  - use the axes provided in the Student Handout, to plot graphs of:
    - vertical reaction, $\mathbf{V_L}$, at the left-hand end of the arch, vs distance **a**;
    - vertical reaction, $\mathbf{V_R}$, at the right-hand end of the arch, vs distance **a**;
    - horizontal thrust **H** vs distance **a**.
  - draw smooth curves through your experimental points.
  - calculate theoretical values of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** for the values of distance **a** used in the experiment and add them to the results table.
  - plot these values on the graph and draw straight lines through them.

Challenge:

- For parts A and B, compare the theoretical and experimental curves and comment on the comparison in the Student Handout.

Worksheet 2
Effect of UDL

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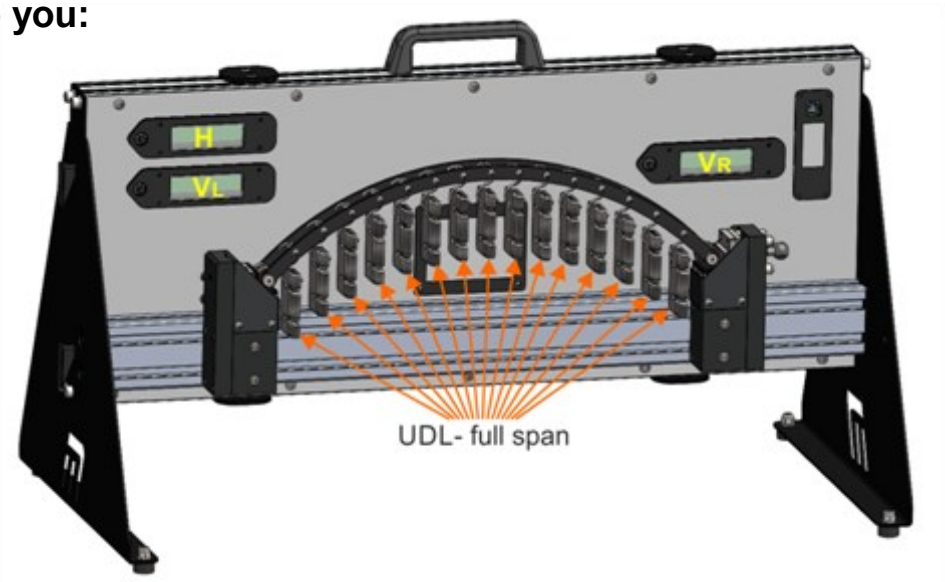
Two and Three Pinned Arch

Worksheet 1 examined the effects of a point load, i.e. one that is concentrated at a particular location.
A distributed load is spread out along the length of the arch. In reality, most loads are distributed, e.g. forces due to water pressure on the dam shown in the photograph.
In a uniformly distributed load, (UDL), the force per unit length on the structure is the same at every point.

A. Two-pinned arch

Over to you:



- Set up the hardware as shown, with the arch touching the left-hand support.
- Turn the support position adjustment to move the right-hand support inwards until it touches the other end of the arch, indicated by a change in the reading on one of the LCD's.
- Press the reset button on each LCD display to initialise the readings.
- Suspend fifteen loads of 20g using S-hooks, to represent a uniformly distributed load.
- Remember to touch the crown of the arch gently to overcome the sensitivity issue.
- As before, observe the readings $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** and record them in the first row of the table 3 in the Student Handout.
- Add an extra 20g load to each of the S-hooks to give a total load of 600g.
- Record the values of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** in the second row of table 3.
- Continue in this way until each S-hook is carrying 100g, i.e. a total load of 1500g.

Worksheet 2
Effect of UDL

MATRIX | STRUCTURES

Two and Three Pinned Arch

So what:

- Complete the second column of table 3 in the Student Handout by calculating the load per unit length, **w**, for each UDL investigated.
- Use your results to plot graphs showing how $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** vary with **w** (on the same axes.) Draw straight lines through the experimental points.
- It can be shown that the vertical reaction forces $\mathbf{V_L}$ and $\mathbf{V_R}$ are given by:

$$\mathbf{V_L} = \mathbf{V_R} = (\mathbf{w} \times \mathbf{L}) / 2$$

and horizontal thrust **H** by

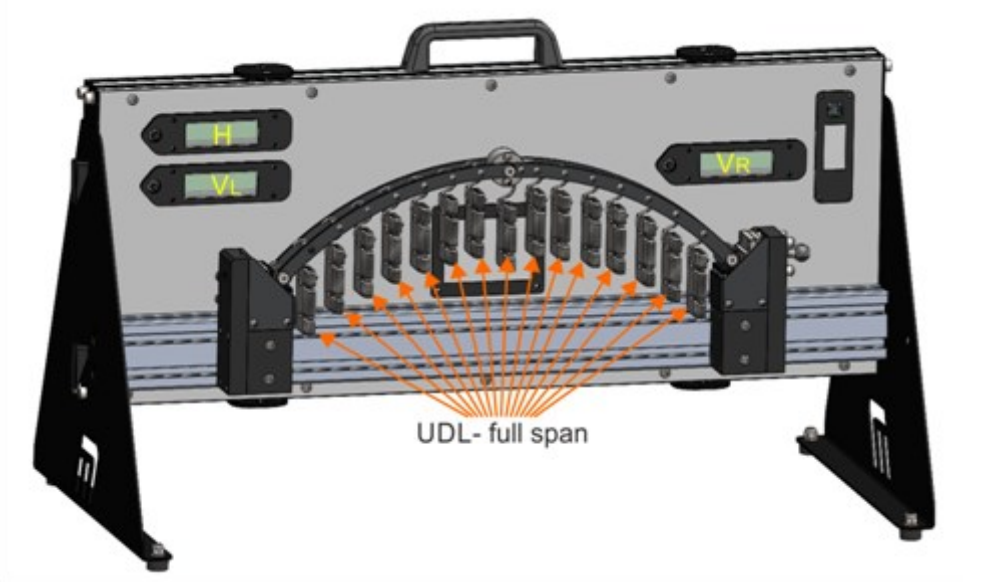
$$\mathbf{H} = \frac{\mathbf{w} \times \mathbf{L}^2}{8\mathbf{r}}$$

These predict that graphs of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** against **w** should be linear.

- Use these to calculate theoretical values of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** and add them to the table 3.
- Plot them on the graph and draw straight lines through them.

B. Three-pinned arch

Over to you:



- Replace the two-pinned arch with the three-pinned arch, keeping the right-hand support in the same position.
- Press the reset button on each LCD display to initialise the readings.
- Repeat the same procedure as in part A, suspending fifteen loads of 20g, to represent a UDL and recording the resulting values of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** in table 4 in the Student Handout.
- As before, progressively increase the suspended loads until each S-hook is carrying 100g, record the resulting values of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** in the table.

Worksheet 2
Effect of UDL

MATRIX | STRUCTURES
Two and Three Pinned Arch

So what:

- Complete the second column of table 4 in the Student Handout by calculating the load per unit length, **w**, for each UDL.
- Use your results to plot graphs showing how $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** vary with **w** (on the same axes.) Draw straight lines through the experimental points.
- As before, the vertical reaction forces $\mathbf{V_L}$ and $\mathbf{V_R}$ are given by:

$$\mathbf{V_L} = \mathbf{V_R} = (\mathbf{w} \times \mathbf{L}) / 2$$

and the horizontal thrust, **H**, by:

$$\mathbf{H} = \frac{\mathbf{w} \times \mathbf{L}^2}{8\mathbf{r}}$$

- Use these to calculate theoretical values of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** and add them to the table 4.
- Plot them on the graph and draw straight lines through them.

Challenge:

1. Since the arch is in static equilibrium, all vertical forces cancel out, all horizontal forces cancel out and the total moment of all forces about any point on the bridge is zero.
   In the Student Handout, show how this information leads to the formula for $\mathbf{V_L}$,$\mathbf{V_R}$ and **H**.
2. Consider whether the weight of the arch itself or the weight of the S-hooks is significant.
   Comment on this in the Student Handout.

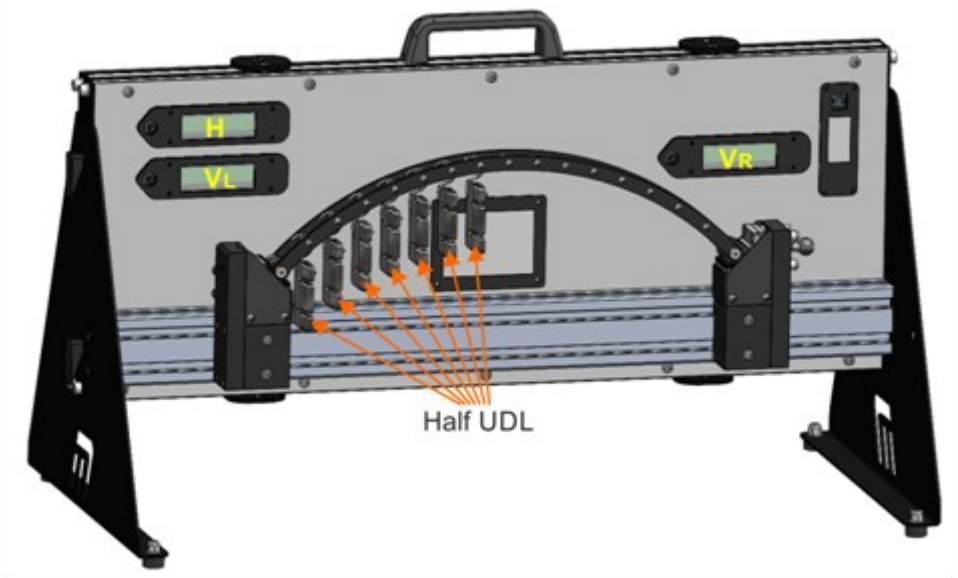
Worksheet 3
Effect of half UDL

MATRIX | STRUCTURES
Two and Three Pinned Arch

Loads carried by arch bridges are not necessarily static. Nor are they point loads.
Sometimes, they are distributed but act only on part of the structure.
This activity examines a load of this kind, known as a half-UDL.

A. Two-pinned arch

Over to you:



- Set up the hardware, with the two-pinned arch placed as before.
- Press the reset button on each LCD display to initialise the readings.
- Add seven loads of 20g from S-hooks as shown, representing a half UDL.
- Touch the crown of the arch gently to overcome the sensitivity issue.
- Take readings of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** and record them in the first row of table 5 in the Student Handout.
- Add an extra 20g load to each of the S-hooks, giving a total load of 280g.
- Record the values of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** in the second row of table 5.
- Continue in this way until each S-hook is carrying 100g, i.e. a total load of 700g.

Worksheet 3
Effect of half UDL

MATRIX STRUCTURES

Two and Three Pinned Arch

So what:

- Complete the second column of table 5 by calculating the load per unit length, **w**, for each half UDL investigated. In this case, **w** = load / 200.
- For the half UDL, the vertical reaction forces $\mathbf{V_L}$ and $\mathbf{V_R}$ are given by:

$$\mathbf{V_L} = \frac{3\,(\mathbf{w} \times \mathbf{L})}{8} \qquad \text{and} \qquad \mathbf{V_R} = \frac{\mathbf{w} \times \mathbf{L}}{8}$$

and horizontal thrust, **H**, by: $$\mathbf{H} = \frac{\mathbf{w} \times \mathbf{L}^2}{16\mathbf{r}}$$

Once again, these predict straight line graphs of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** against **w**.

On the same axes, plot three graphs showing how $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** vary with **w** using your measurements and draw straight lines through the experimental points.

- Use the formulae to calculate theoretical values of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H**. Add them to the table 5.
- Plot them on the graph and draw straight lines through them.

B. Three-pinned arch

Over to you:

- Replace the two-pinned arch with the three-pinned arch, keeping the right-hand support in the same position.
- Press the reset button on each LCD display to initialise the readings.
- Repeat the same procedure as in part A, suspending seven loads of 20g, as a half UDL and recording the resulting values of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** in table 6 in the Student Handout.
- As before, progressively increase the suspended loads until each S-hook is carrying 100g, record the resulting values of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** in the table.

Worksheet 3
Effect of half UDL

So what:

- Complete the second column of table 6 by calculating the load per unit length, **w**, for each half UDL investigated. In this case, **w** = load / 200.
- On the same axes, plot three graphs showing how $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** vary with **w** using your measurements and draw straight lines through the experimental points.
- Use the formulae given on the previous page to calculate theoretical values of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** and add them to the table 6.
- Plot them on the graph and draw straight lines through them.

Challenge:

Compare the performance of two-pinned and three-pinned arches when carrying point loads and UDLs, using:

- your findings from the three worksheets;
- research on the internet and elsewhere.

Write the results of your comparison in the Student Handout.

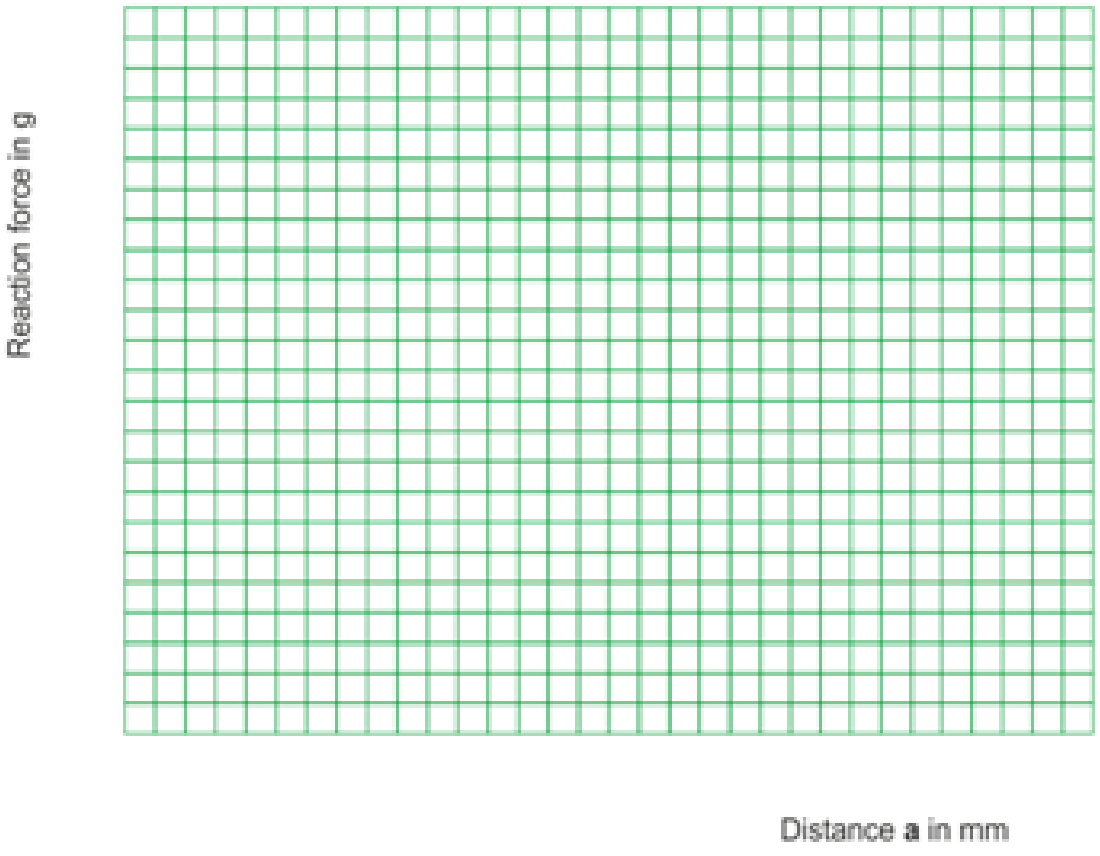
Student Handout

Worksheet 1 - Effect of position of point load

A. Two-pinned arch

Table 1

a mm	Experimental			Calculated		
	H g	$\mathbf{V_L}$ g	$\mathbf{V_R}$ g	**H** g	$\mathbf{V_L}$ g	$\mathbf{V_R}$ g
50						
75						
100						
125						
150						
175						
200						
225						
250						
275						
300						
326						
350						

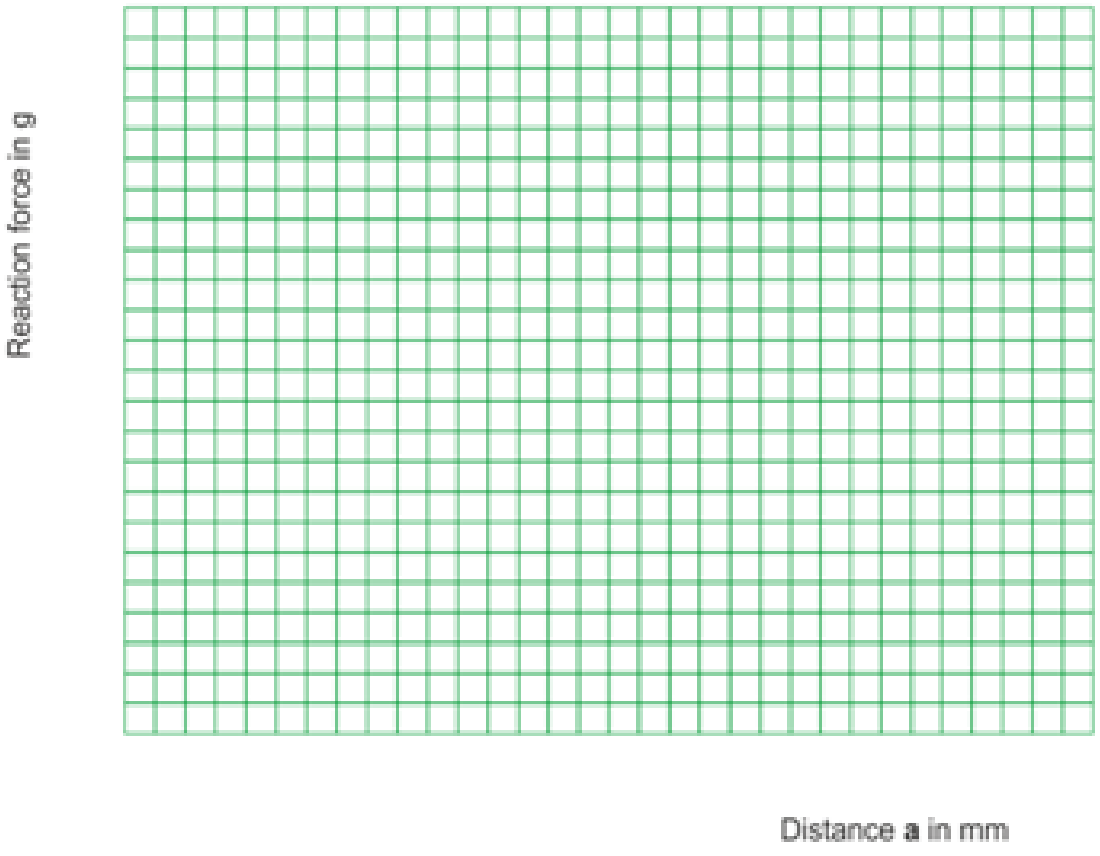


Worksheet 1 - Effect of position of point load

B. Three-pinned arch

Table 2

a mm	Experimental			Calculated		
	H g	$\mathbf{V_L}$ g	$\mathbf{V_R}$ g	**H** g	$\mathbf{V_L}$ g	$\mathbf{V_R}$ g
50						
75						
100						
125						
150						
175						
200						
225						
250						
275						
300						
326						
350						



MATRIX | STRUCTURES

Two and Three Pinned Arch

Worksheet 1 - Effect of position of point load

To calculate theoretical values for reaction and thrust:

vertical reactions are given by

$$V_L = \frac{W \,.\, (L - a)}{L} \qquad V_R = \frac{W \,.\, a}{L}$$

and the horizontal thrust is given by

$$H = 5 \,.\, \frac{W \,.\, a \,(L^3 + a^3 - 2\, L \,.\, a^2)}{8 \,.r \,.L^3}$$

Comment on the comparison between experimental and theoretical values:

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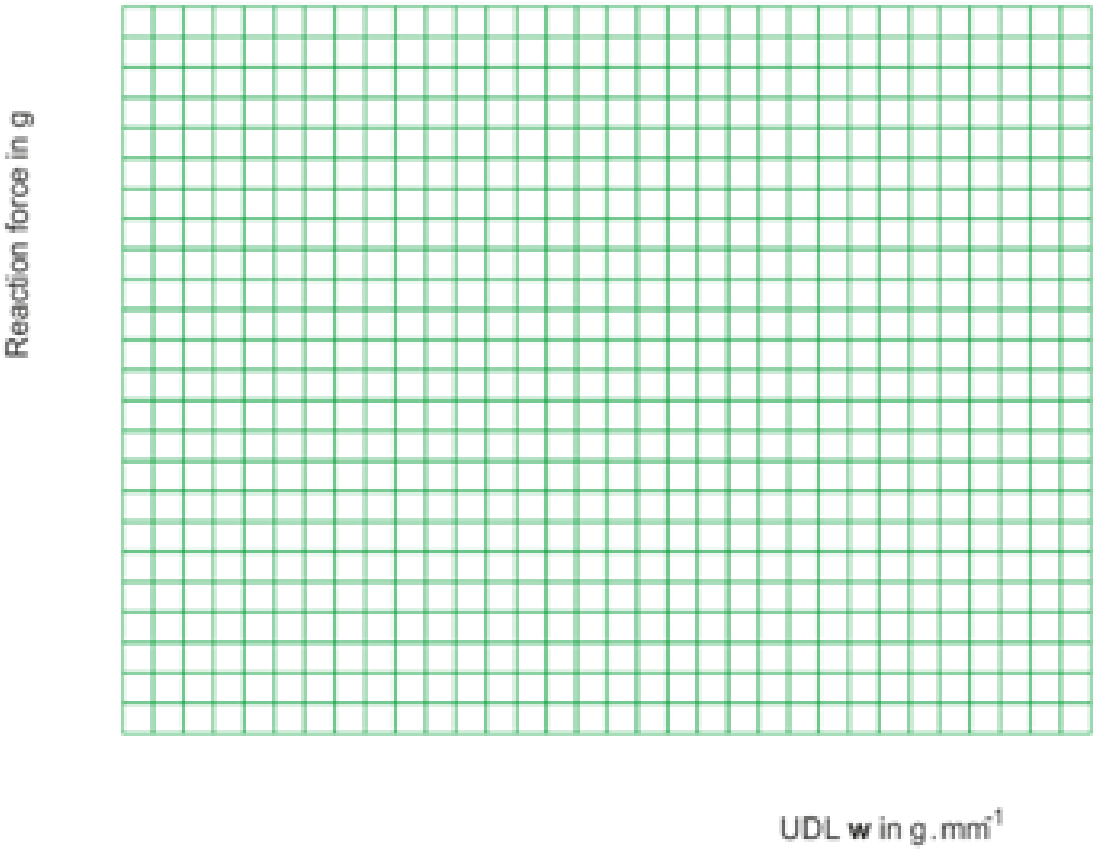
Worksheet 2 - Effect of UDL

A. Two-pinned arch

Table 3

Total load g	**w** g.mm^{-1}	Experimental			Calculated		
		H g	$\mathbf{V_L}$ g	$\mathbf{V_R}$ g	**H** g	$\mathbf{V_L}$ g	$\mathbf{V_R}$ g

Graphs of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** vs **w**:



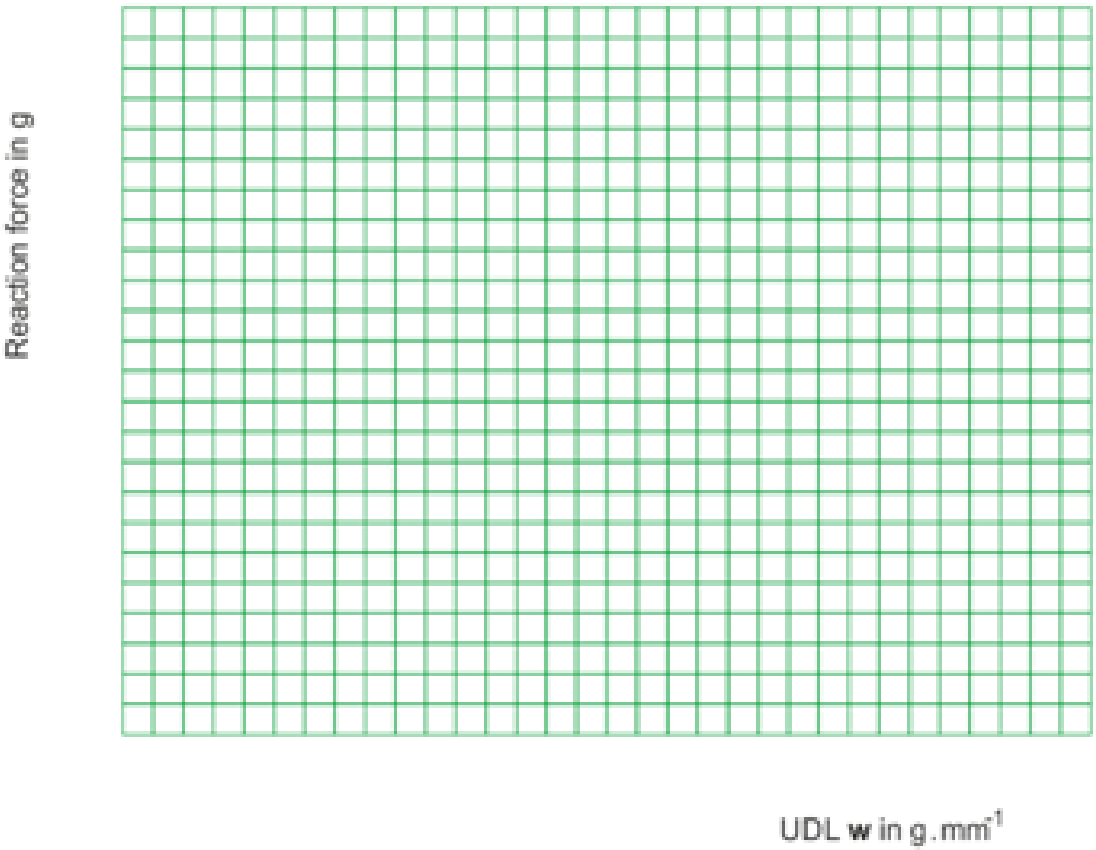
Worksheet 2 - Effect of UDL

B. Three-pinned arch

Table 4

Total load g	**w** g.mm^{-1}	Experimental			Calculated		
		H g	$\mathbf{V_L}$ g	$\mathbf{V_R}$ g	**H** g	$\mathbf{V_L}$ g	$\mathbf{V_R}$ g

Graphs of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** vs **w**:



Worksheet 2 - Effect of UDL

Challenge:

- Obtain formulae to allow you to calculate theoretical values for $\mathbf{V_L}$, $\mathbf{V_R}$ and **H**:

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2. Consider whether the weight of the arch itself or the weight of the S-hooks is significant.

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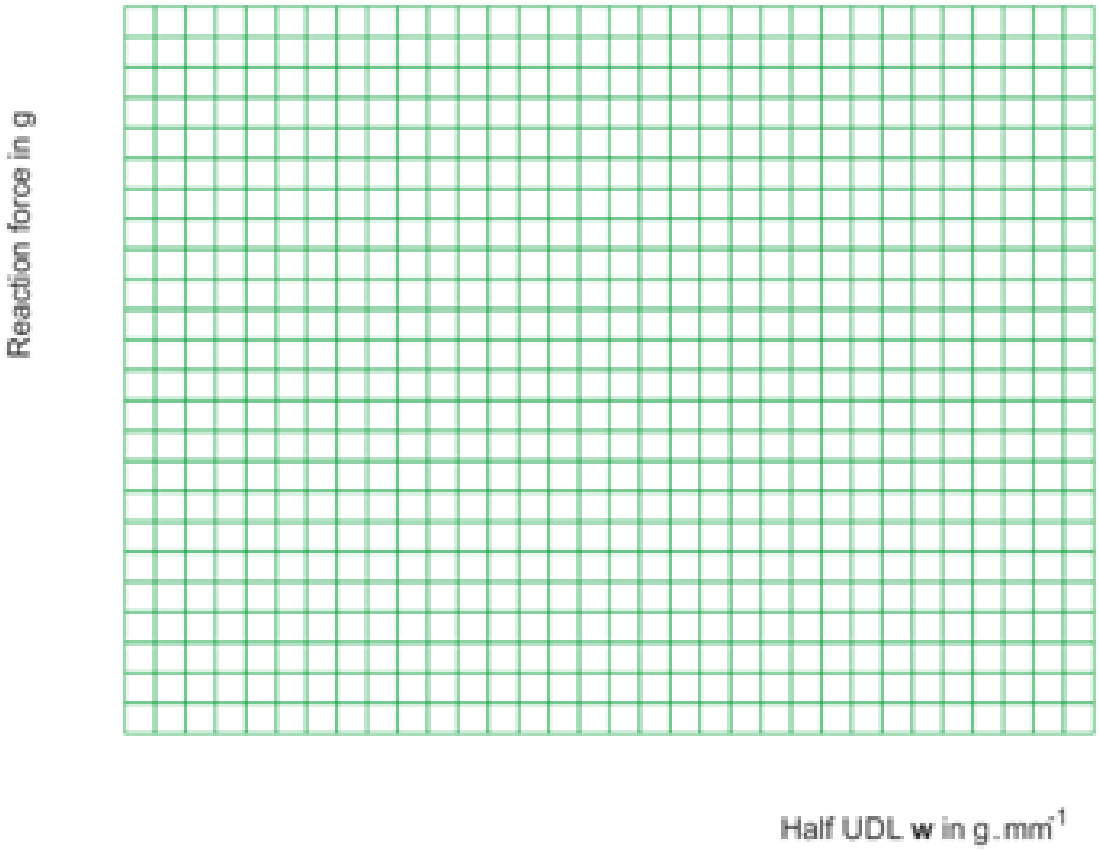
Worksheet 3 - Effect of half UDL

A. Two-pinned arch

Table 5

Total load g	**w** g.mm^{-1}	Experimental			Calculated		
		H g	$\mathbf{V_L}$ g	$\mathbf{V_R}$ g	**H** g	$\mathbf{V_L}$ g	$\mathbf{V_R}$ g

Graphs of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** vs **w**:



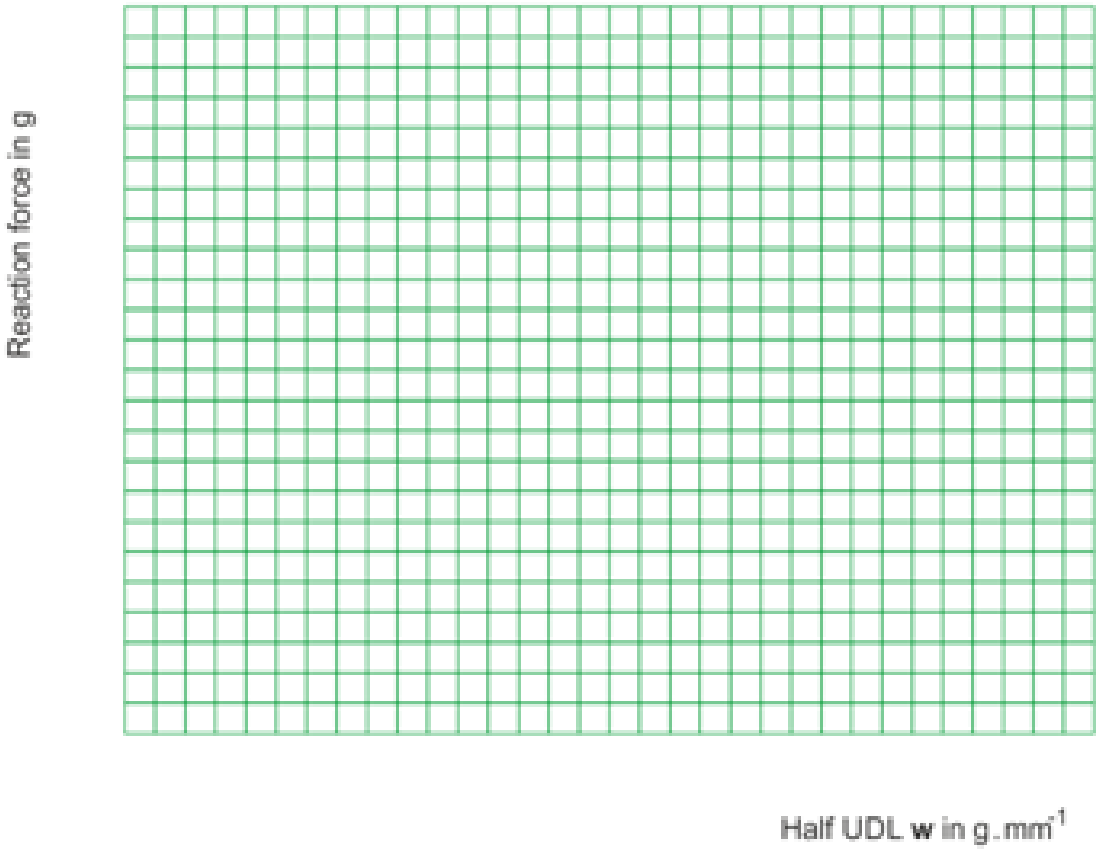
Worksheet 3 - Effect of half UDL

B. Three-pinned arch

Table 6

Total load g	**w** g.mm^{-1}	Experimental			Calculated		
		H g	$\mathbf{V_L}$ g	$\mathbf{V_R}$ g	**H** g	$\mathbf{V_L}$ g	$\mathbf{V_R}$ g

Graphs of $\mathbf{V_L}$, $\mathbf{V_R}$ and **H** vs **w**:



Challenge:

Comment on the comparison between two-pinned and three-pinned arches: