

# **MATRIX** | FUNDAMENTAL MECHANICS

## Dynamics Plus



**MATRIX**

CP0978

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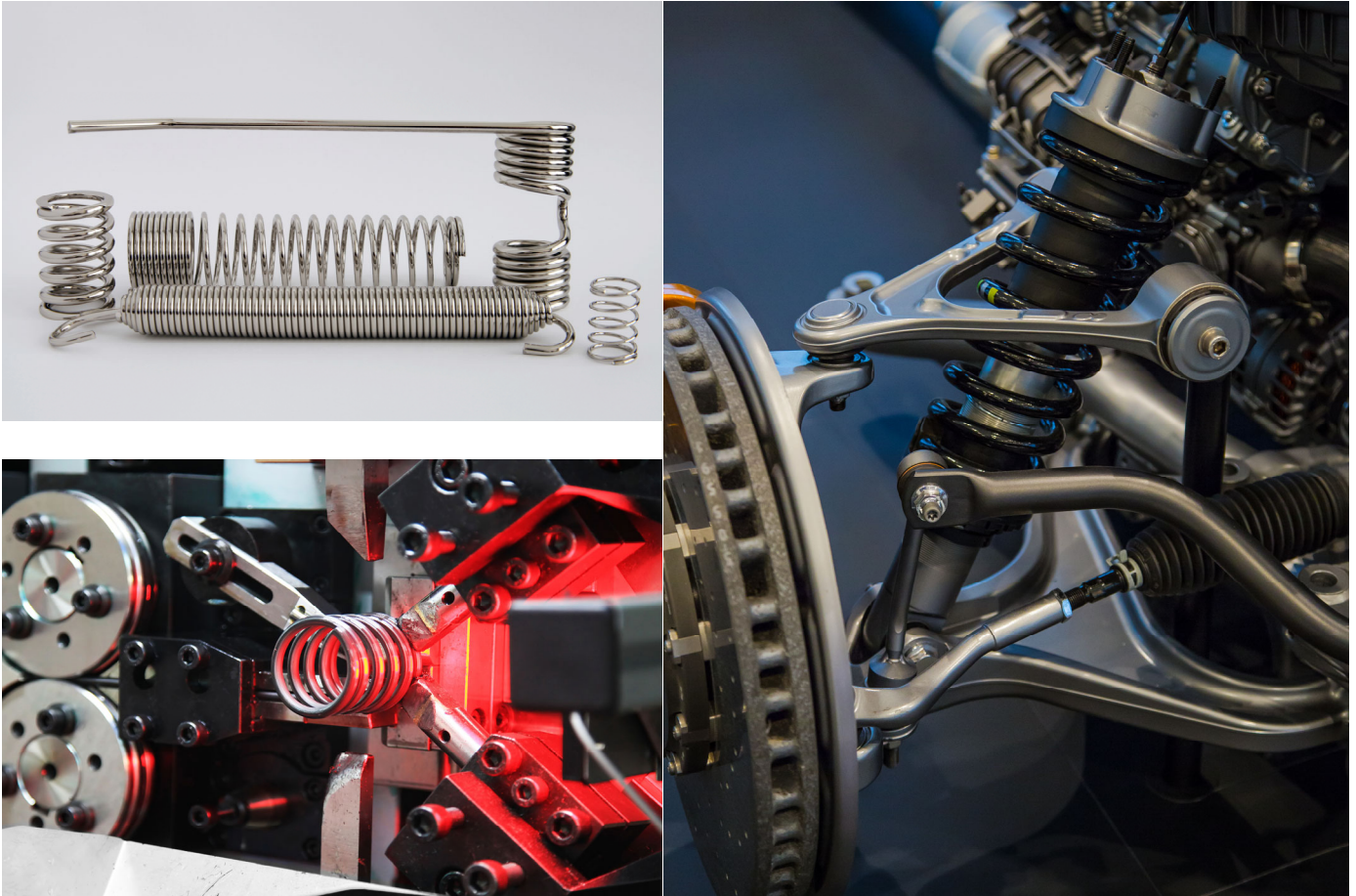
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# Contents

Chapter 1—Introduction to Springs		3
Worksheet 1	Single compression spring	4
Worksheet 2	Compression springs in parallel	6
Worksheet 3	Compression springs in series	8
Worksheet 4	Single extension spring	10
Worksheet 5	Extension springs in series	12
Worksheet 6	Extension springs in parallel	14
Chapter 2 - Introduction to SMH		16
Worksheet 7	Oscillating Springs	18
Worksheet 8	Simple Pendulum	21
Worksheet 9	Kater's Pendulum	23
Worksheet 10	Triflar Pendulum	26
Chapter 3—Introduction to Centrifugal Forces		28
Worksheet 11	Preliminary Centrifugal tests	31
Worksheet 12	Measuring Centrifugal Forces	33
Student Handout		36

# Introduction

Springs, usually made of steel, distort when subjected to a load and then normally return to their original shape once the load is removed.



They come in a variety of shapes and sizes - helical, torsion, spiral, leaf, disc and etc. Huge springs in their foundations protect some buildings against earthquakes. Tiny ones are found in medical implants such as heart pacemakers. They store energy, absorb energy, hold components in place, create motion, measure force ...

Springs are found in all manner of everyday devices:

- compression springs and leaf springs used in car suspension to cushion rough terrain;
- hair springs used to store and release energy in analogue clock mechanisms;

helical springs used in dynamometers to measure force ...

# Worksheet 1

## Single compression spring

Compression springs are used in a wide variety of devices, from pogo sticks to ball pens to car suspensions.

All exhibit a property known as *elasticity*, the ability of a body to return to its original shape and size once a load is removed.

Choosing the right material for them is important. Common materials used include high carbon steel and nickel and



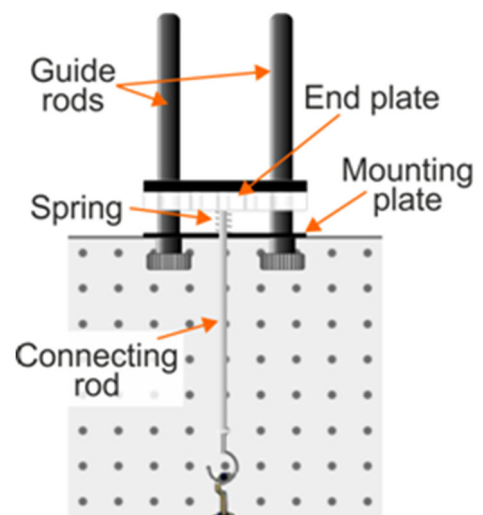
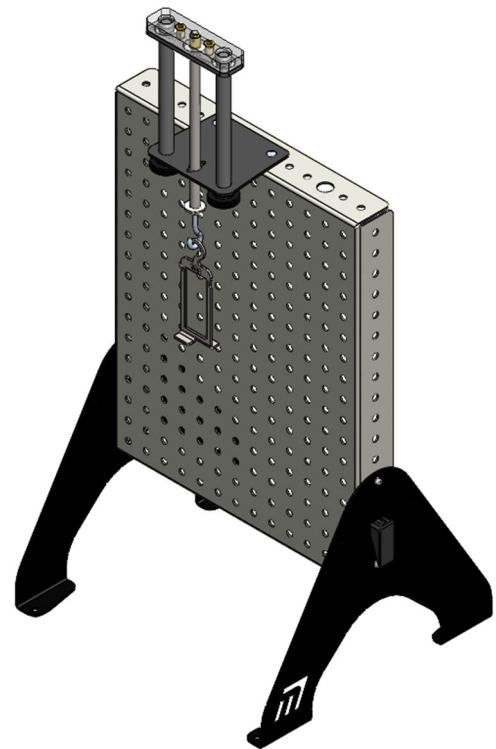
Over to you:

This investigation looks at the behaviour of a spring when compressed.

Set up the system shown in the diagram, with the baseboard mounted in 'portrait' orientation.

For the top assembly:

- Use thumbscrews to fasten the guide rods to the mounting plate with .
- Attach the mounting plate to the baseboard with thumbscrews.
- Pass the end of the connecting rod through the hole
- in the mounting plate.
- Place the spring on the connecting rod and screw the rod into the end plate.
- Lower the end plate over the guide rods.



# Worksheet 1

## Single compression spring

### Over to you.....

- Place the ruler on the baseboard so that it is vertical and behind the connecting rod.
- Choose a suitable reference point on the connecting rod, such as the washer at the end.
- In the Student Handout, note the ruler scale reading level with this reference point.
- Suspend a load of 60g on a hanger from the hook at the bottom of the connecting rod. (The empty weight hanger has a mass of 20g.)
- Note the new ruler scale reading level with the reference point.
- Subtract the initial scale reading from this new one to obtain the spring deflection (change in length).
- Record this in the table in the Student Handout.
- Increase the load by 60g and measure the new spring deflection in the same way.
- Continue this process until the hanger carries a total load of 480g.

### So what:

- Convert the load readings from grams (i.e. the masses of the loads) into equivalent weights using the relationship  $F = ma$ .

$$a = 9.81 \text{ m/s}^2$$

- Using your results, plot a graph of spring deflection versus added weight. The result should be a straight line trace, passing through the origin. Use your points as a guide to position this straight line.
- Calculate the gradient of that line to give the spring constant for the spring used in the experiment and record it in the Student Handout.
- Repeat this procedure for the other springs provided in the kit.

### Challenge:

Examine sources of error in your readings and suggest ways of reducing them.

Plot  $F$  on the  $y$  axis and the deflection on the  $x$  axis. This means that your spring constant is the gradient of the line.

## Worksheet 2

### Compression springs in parallel

Combining several compression springs in parallel can:

- increase their total load bearing capacity;
- spread the effect of their support over a larger area;
- build in some redundancy in the event of one of the springs failing.

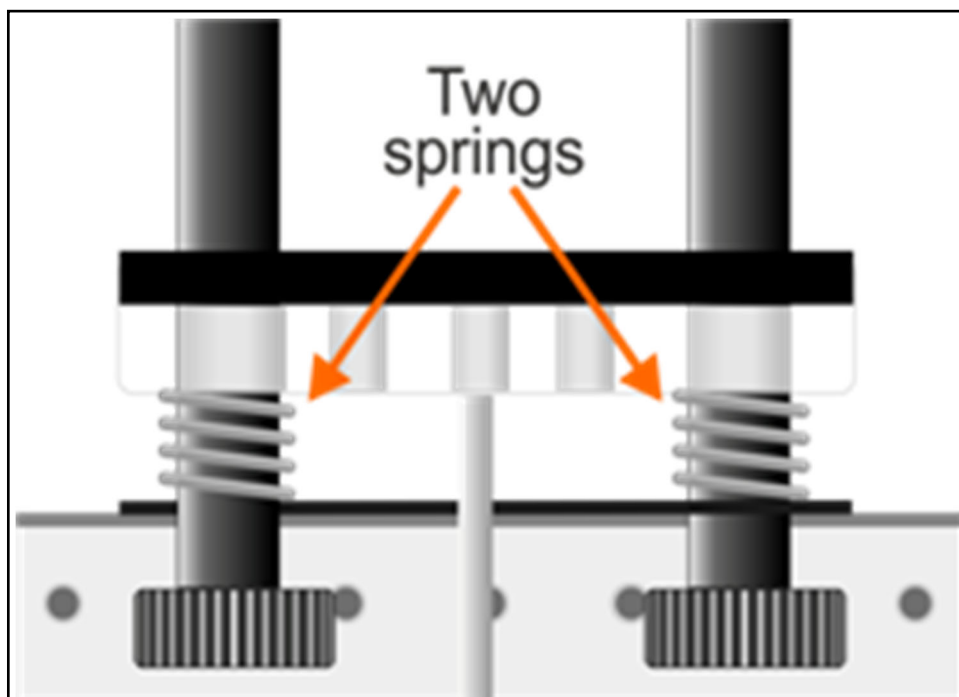


#### Over to you:

Set up the system shown in the diagram.

For the top assembly:

- Unscrew the connecting rod from the end plate.
- Lift off the end plate and springs.
- Place the springs on the guide rails, as shown opposite.
- Screw the rod back into the end plate.



# Worksheet 2

## Compression springs in parallel

### Over to you.....

- Place the ruler on the baseboard so that it is vertical and behind the connecting rod.
- Choose a suitable reference point on the connecting rod, such as the washer at the end.
- In the Student Handout, note the ruler scale reading level with this reference point.
- Suspend a load of 60g on a hanger from the hook at the bottom of the connecting rod. (The empty weight hanger has a mass of 20g.)
- Note the new ruler scale reading level with the reference point.
- Subtract the initial scale reading from this new one to obtain the spring deflection (change in length).
- Record this in the table in the Student Handout.
- Increase the load by 60g and measure the new spring deflection in the same way.
- Continue this process until the hanger carries a total load of 480g.

### So what:

- Convert the load readings from grams (i.e. the masses of the loads) into equivalent weights using the relationship  $F = ma$ .

$$a = 9.81 \text{ m/s}^2$$

- Using your results, plot a graph of spring deflection versus added weight. Use your points as a guide to position the straight line trace.
- Calculate the gradient of this straight line trace to give the spring constant for the spring used in the experiment and record it in the Student Handout.
- Repeat this procedure for the other springs provided in the kit.

### Challenge:

Compare the performance of this system, (two springs in parallel) with that investigated in worksheet 1.

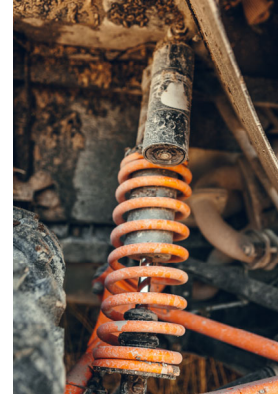
# Worksheet 3

## Compression springs in series

Sometimes, several springs may operate in series, either connected physically, end-to-end, or separated by a rigid part of the machinery.

One application is the 'progressive rate' spring, having a spring constant that varies along its length.

These change their resistance as they are compressed and are often used in vehicle suspension systems.



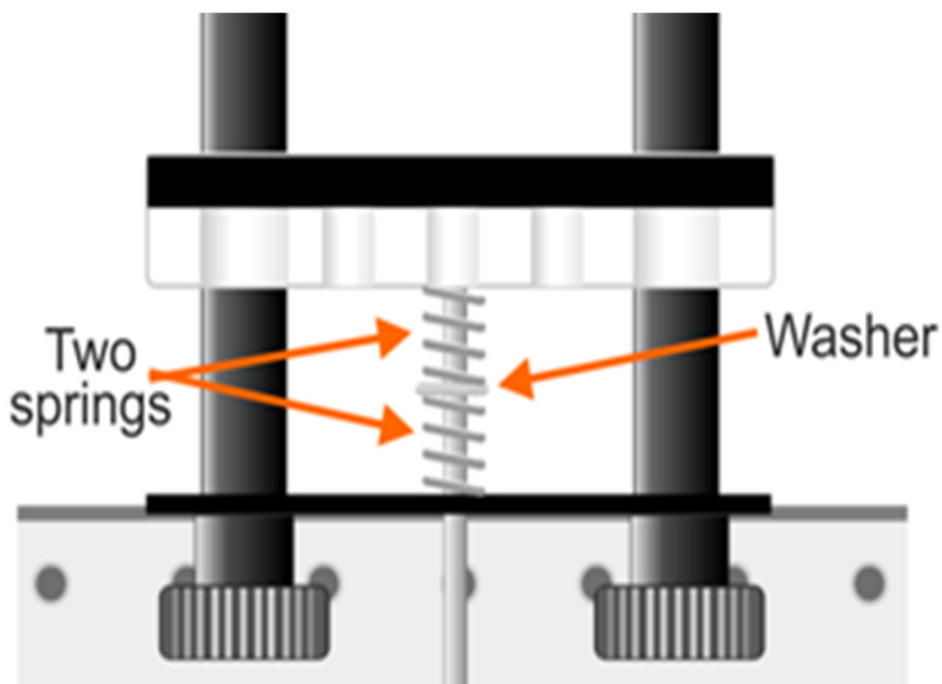
### Over to you:

In this worksheet, two identical compression springs operate in series, separated by a washer.

Set up the system shown in the diagram.

For the top assembly:

- Unscrew the connecting rod from the end plate.
- Lift up the end plate and place a washer on the rod on top of the original spring.
- Place a second identical spring on top of the washer, (i.e. in series with the first spring).
- Screw the rod back into the end plate.





# Worksheet 3

## Compression springs in series

### Over to you.....

The test procedure follows that used in worksheet 1:

- Place the ruler vertically behind the connecting rod.
- Choose a suitable reference point on the connecting rod, such as the disc at the end.
- In the Student Handout, note the ruler scale reading level with this reference point.
- First of all, suspend a load of 60g on a hanger from the hook at the bottom of the connecting rod. (The empty weight hanger has a mass of 20g.)
- Note the new ruler scale reading level with the reference point.
- Subtract the initial scale reading to obtain the resulting spring deflection.
- Record this in the table in the Student Handout.
- Increase the load by 60g and measure the new spring deflection in the same way.
- Continue this process until the hanger carries a total load of 480g.

### So what:

- Convert the load readings from grams (i.e. the masses of the loads) into equivalent weights using the relationship  $F = ma$ .

$$a = 9.81 \text{ m/s}^2$$

- Using your results, plot a graph of spring deflection versus added weight. Again, the result should be a straight line trace, passing through the origin. Use your points as a guide to position this straight line.
- Calculate the gradient of this line to give the spring constant for the spring used in the experiment and record it in the Student Handout.
- Repeat this procedure for the other springs provided in the kit.

### Challenge:

Compare the performance of this system, (two springs in series) with that investigated in worksheet 1.

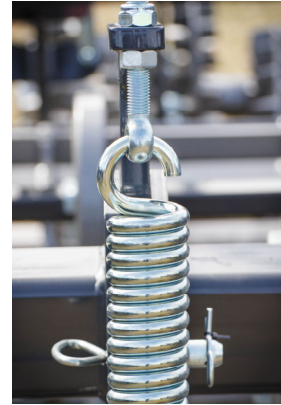
# Worksheet 4

## Single extension spring

A force meter relies on the behaviour of a single spring, anchored at its upper end to the body of the meter, with a hook connected at the lower end.

It measures the weight of an object by opposing the gravitational force acting on the body with a force due to the extended spring.

It relies on Hooke's law, which states that the force needed to extend the spring changes linearly with distance. As a result, the scale markings on the meter body are equally spaced.

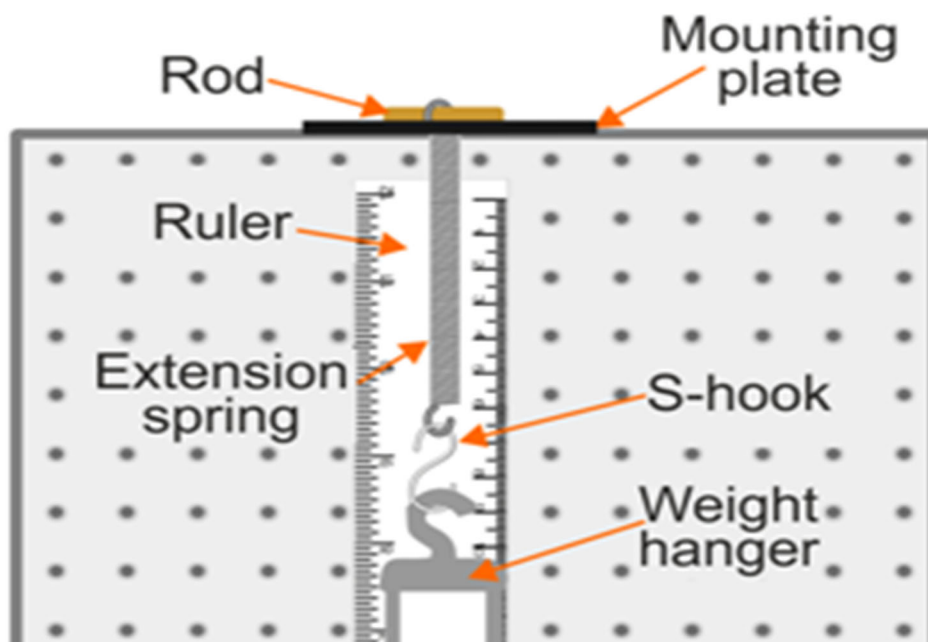


### Over to you:

In this arrangement, an extension spring hangs from the mounting plate, as shown in the diagram.

For the top assembly:

- Unscrew and remove the guide rods from the mounting plate.
- Unscrew the connecting rod from the end plate and remove both.
- Push the extension spring through the hole in the mounting plate and slide the metal rod through the loop at the top end of the spring to secure it.
- Suspend a weight hanger from a S-hook looped through the other end.



# Worksheet 4

## Single extension spring

### Over to you.....

Investigate what happens to the spring when various loads are added to the weight hanger, as follows:

- Place the ruler vertically behind the spring.
- Choose a suitable reference point, such as the end of the hook on the weight hanger.
- In the Student Handout, note the ruler scale reading level with this reference point.
- First of all, suspend a load of 100g on a hanger from the hook at the bottom of the connecting rod. (Remember - the empty weight hanger has a mass of 20g.)
- Note the new ruler scale reading level with the reference point.
- Subtract the initial scale reading to obtain the resulting spring deflection.
- Record this in the table in the Student Handout.
- Increase the load by 100g and measure the new spring deflection in the same way.
- Continue this process until the hanger carries a total load of 500g.

### So what:

- Convert the load readings from grams (i.e. the masses of the loads) into equivalent weights using the relationship  $F = ma$ .

$$a = 9.81 \text{ m/s}^2$$

- Using your results, plot a graph of spring deflection versus added weight. As before, the result should be a straight line trace, passing through the origin. Use your points as a guide to position this straight line.
- Calculate the gradient of that line to give the spring constant for the spring used in the experiment and record it in the Student Handout.

### Challenge:

Explain what would happen if two force meters were hung, one below the other and a weight suspended from the lower one.

# Worksheet 5

## Extension springs in series

Within a series setup, springs are connected one after the other.

Both experience the full applied force. Both experience an extension of their length.

The result is a greater overall extension and improved elasticity, an attribute beneficial in designs that demand high levels of operational flexibility.

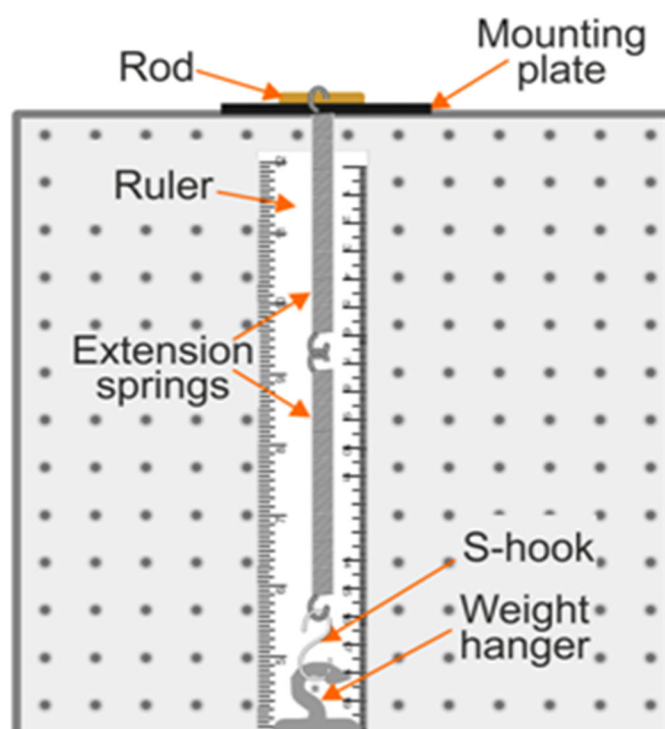


### Over to you:

In this arrangement, an extension spring hangs from the mounting plate, as shown in the diagram.

For the top assembly:

- Link two extension springs together.
- Push the end of one through the hole in the mounting plate and slide the metal rod through the loop at the top end of the spring to secure it.
- Suspend a weight hanger from a S-hook looped through the end of the lower spring.



# Worksheet 5

## Extension springs in series

### Over to you.....

Investigate what happens to the spring when various loads are added to the weight hanger, as follows:

- Place the ruler vertically behind the spring.
- Choose a suitable reference point, such as the end of the hook on the weight hanger.
- In the Student Handout, note the ruler scale reading level with this reference point.
- First of all, suspend a load of 100g on a hanger from the hook at the bottom of the connecting rod. (Remember - the empty weight hanger has a mass of 20g.)
- Note the new ruler scale reading level with the reference point.
- Subtract the initial scale reading from this new one to obtain the resulting spring deflection.
- Record this in the table in the Student Handout.
- Increase the load by 100g and measure the new spring deflection in the same way.
- Continue this process until the hanger carries a total load of 500g.

### So what:

- Convert the load readings from grams (i.e. the masses of the loads) into equivalent weights using the relationship  $F = ma$ .

$$a = 9.81 \text{ m/s}^2$$

- Using your results, plot a graph of spring deflection versus added weight. Use your points as a guide to position the straight line trace.
- Calculate the gradient of that line to give the effective spring constant for the spring combination used in the experiment and record it in the Student Handout.

### Challenge:

Find out as much as you can about progressive (variable) rate springs.

# Worksheet 6

## Extension springs in parallel

When a load is applied to springs connected in parallel, they share the load and so are not stretched as much as they would be if they were supporting the load on their own.

The combination is 'stiffer'.

For two springs connected in this way, the effective spring constant will be twice that of a single spring.

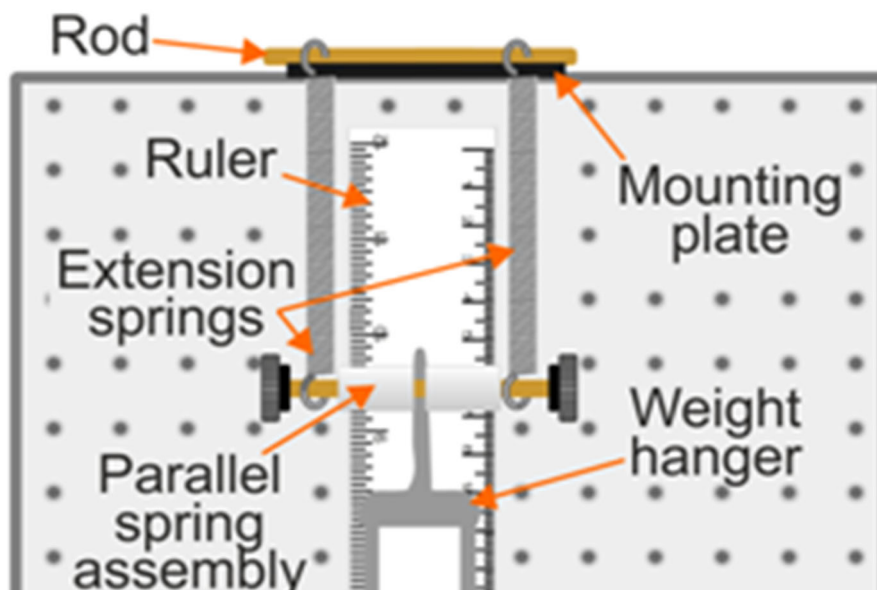


### Over to you:

- In this arrangement, two extension springs, attached to the parallel spring assembly, hang from the mounting plate, as shown in the next diagram.

For the top assembly:

- Attach two extension springs to the parallel spring assembly.
- Push the upper end of each through the hole in the mounting plate and slide the metal rod through the end loops to secure them.
- Check that the two springs hang vertically from the mounting plate.
- Suspend a weight hanger from the mid-point of the parallel spring assembly.



# Worksheet 6

## Extension springs in parallel

### Over to you.....

Investigate what happens when various loads are added to the weight hanger, as follows:

- Place the ruler vertically behind the spring.
- Choose a suitable reference point, such as the end of the hook on the weight hanger.
- In the Student Handout, note the ruler scale reading level with this reference point.
- Place a load of 100g on the spring. (Remember - the empty weight hanger has a mass of 20g.)
- Check that the parallel spring assembly rod is horizontal.
- Note the new ruler scale reading level with the reference point.
- Subtract the initial scale reading to obtain the resulting deflection of the spring.
- Record this in the table in the Student Handout.
- Increase the load by 100g and measure the new spring deflection in the same way.
- Continue this process until the hanger carries a total load of 500g.

### So what:

- Convert the load readings from grams (i.e. the masses of the loads) into equivalent weights using the relationship  $F = ma$ .

$$a = 9.81 \text{ m/s}^2$$

- Using your results, plot a graph of spring deflection versus added weight. Use your points as a guide to position the resulting straight line trace.
- Calculate the gradient of the resulting straight line to give the effective spring constant for the spring combination and record it in the Student Handout.

### Challenge:

Describe one situation where combining springs in parallel would be preferable to using a single spring.

# Chapter 2

## Simple Harmonic Motion

### Periodic Motion

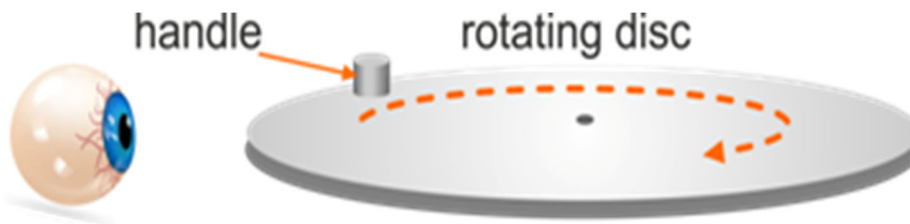
In this, a body oscillates back and forth with the motion repeating in equal time intervals (known as the period). There is no stable equilibrium position and no restoring force.

### Simple Harmonic Motion

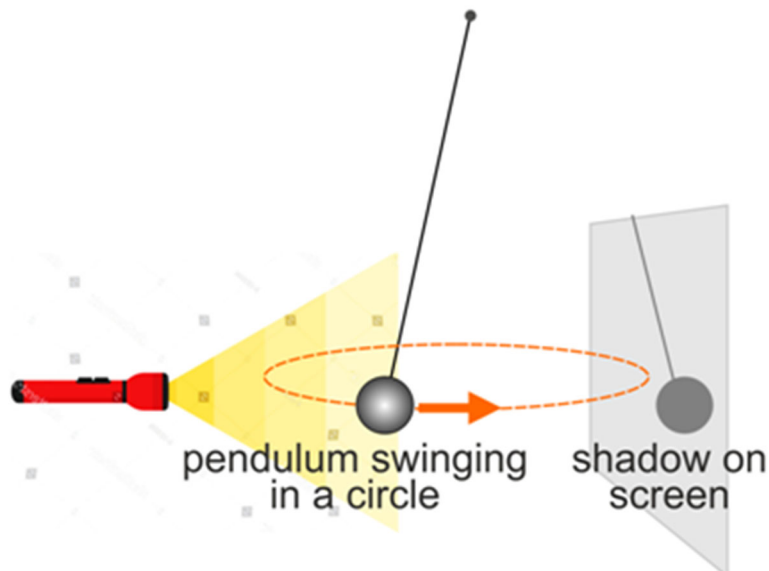
Simple harmonic motion (SHM) is a special case of periodic motion, where there is a restoring force, which is always directed towards the equilibrium position and is directly proportional to the distance from it.

To visualise SHM, you could:

- Spin a disc with a handle on it at steady speed and view it edge-on.  
The movement of the handle simulates SHM.



- Launch a pendulum in a circle and shine a light from the side.  
The shadow of the bob mimics SHM.

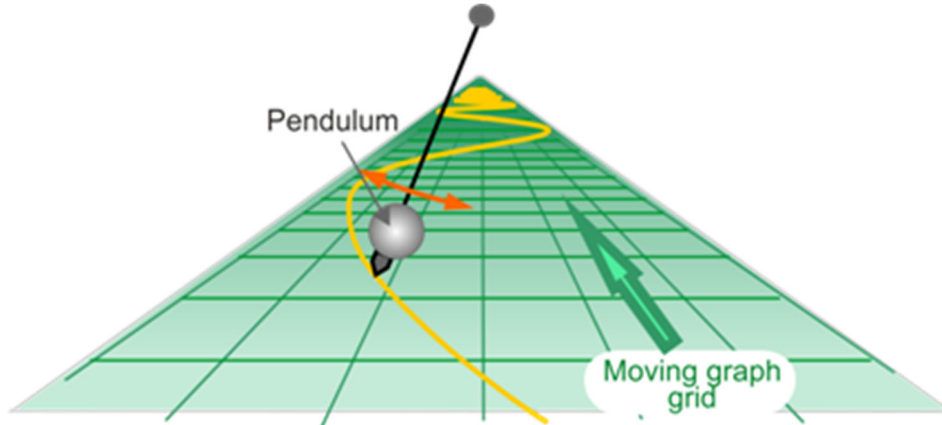




# Chapter 2

## Simple Harmonic Motion

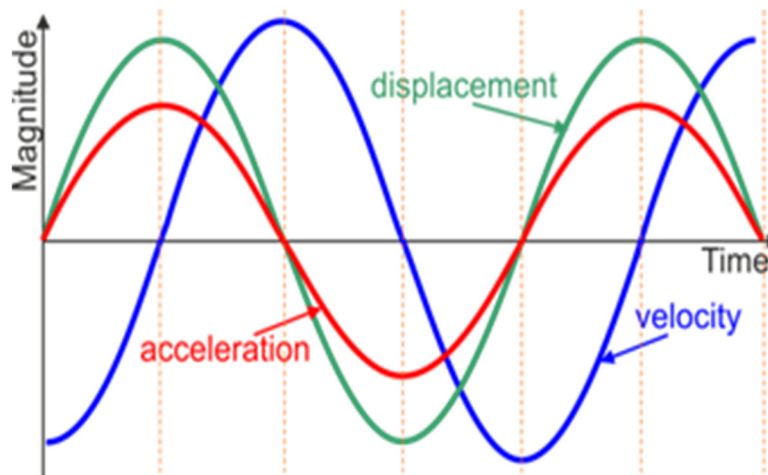
Alternatively, study the diagram below.



A pendulum is swinging from side to side. Its path is marked out on the grid of a graph, which is itself slowly moving into the distance.

The result is a graph of simple harmonic motion.

The next graph gives more information. Worksheet 6 Extension springs in parallel



The green trace shows how the **displacement**, (distance from the equilibrium position,) varies with time.

The blue trace shows the **velocity** of the bob.

Notice that it is zero at maximum displacement and maximum at zero displacement.

The red trace shows its acceleration.

Even though the bob is not actually moving, the **acceleration** is maximum at its maximum displacement, (since that is where the restoring force is at its maximum.)

# Worksheet 7

## Oscillating springs

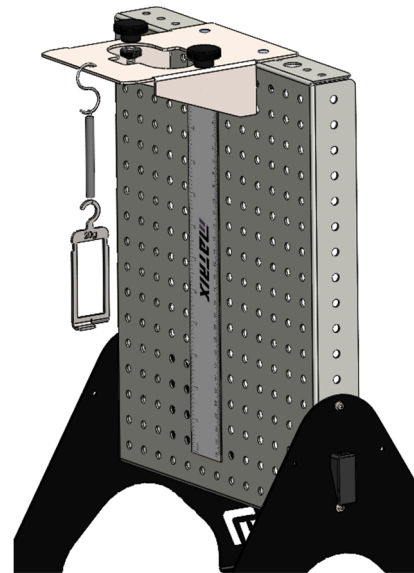
A weight hanging on a spring can bounce up and down repeatedly as a result of the interplay between gravity and the elastic force in the spring. The gravitational force is constant. The elastic force depends on the weight's displacement from the equilibrium position.



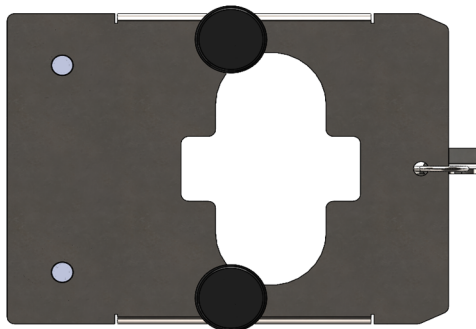
When in the equilibrium position, the gravitational force and the elastic force of the spring balance each other. When the weight is above this point the elastic force is smaller than gravitational force. The resultant is a force directed back towards the equilibrium

### Over to you:

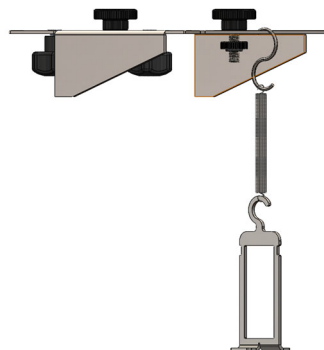
- Set up the system shown in the diagram, with the baseboard mounted in 'portrait' orientation.



For the top assembly, use the mounting plate shown below (underside view).



Top view



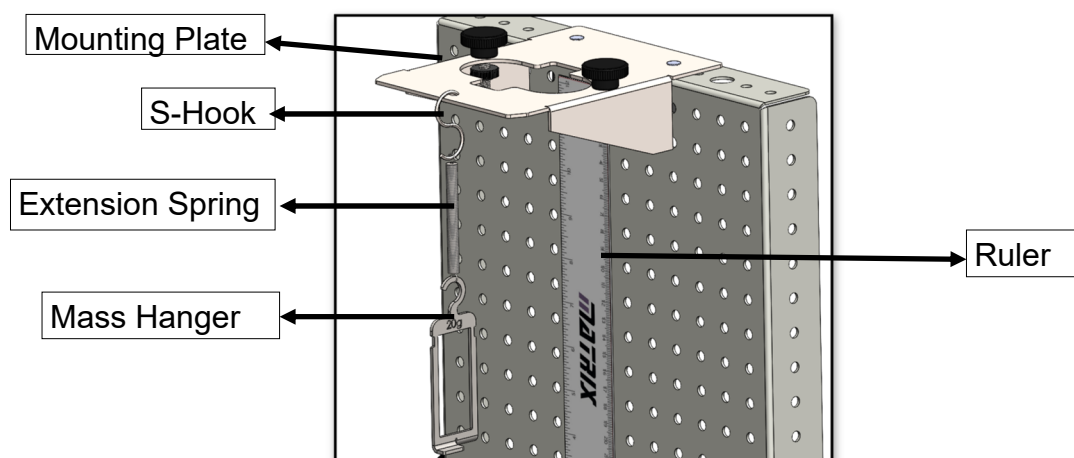
Side view

# Worksheet 7

## Oscillating springs

### Over to you.....

- Undo the frontal thumbscrew and use the S hook provided instead.
- Place S hook through the frontal hole on the mounting plate and attach the extension spring provided at the other end. Attach the mass hanger on the end of the spring as shown below.
- Position the magnetic ruler to read the length of the spring.



#### A. Measure the extension:

- Choose a suitable reference point on the spring / hanger to measure the unloaded length of the spring.
- Hang a mass (total, with hanger) of 200g from the spring and calculate its weight, **W**.
- With the spring motionless, use the reference point to measure its new length.
- Calculate the resulting extension of the spring.
- Record all results in the Student Handout.

#### B. Measure the oscillations:

- Pull the weight vertically down to stretch the spring about 20mm.
- Release it and steady it so that it oscillates vertically.
- Use a stopwatch to time 25 oscillations.
- Divide this time by 25 to calculate the period, **T**, of the oscillation.
- Record these readings in the table in the Student Handout.
- Repeat the procedure described in **A** and **B** using total masses of 300g, 400g, 500g and 600g in turn.

# Worksheet 7

## Oscillating springs

### So what:

- Using your results from part **A** above, plot a graph of spring deflection versus load to give a straight line trace, passing through the origin.
- Use your points as a guide to position this straight line and calculate its gradient, to give the spring constant, **k**, for the spring.
- Record it in the Student Handout.
- For each value of load added, calculate an experimental value of its mass, **m**, using the formula:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

which can be rearranged to give:

$$m = \frac{kT^2}{4\pi^2}$$

### Challenge:

1. Using a total mass of 300g, investigate the effect on the period of oscillation, **T**, of increasing the initial deflection of the load. Summarise your results in the Student Handout.
2. Repeat the investigations described in parts **A** and **B** using a spring with a bigger spring constant. Record your results in table 1B in the Student Handout.

# Worksheet 8

## The simple pendulum

Under the right conditions, a swinging pendulum 'beats' out equal intervals of time. Historically, this provided important time standards in navigation, transport and factories.

So important were they that more complex forms were designed, like that in the smaller picture, that incorporated temperature compensation to allow for thermal expansion and contraction.

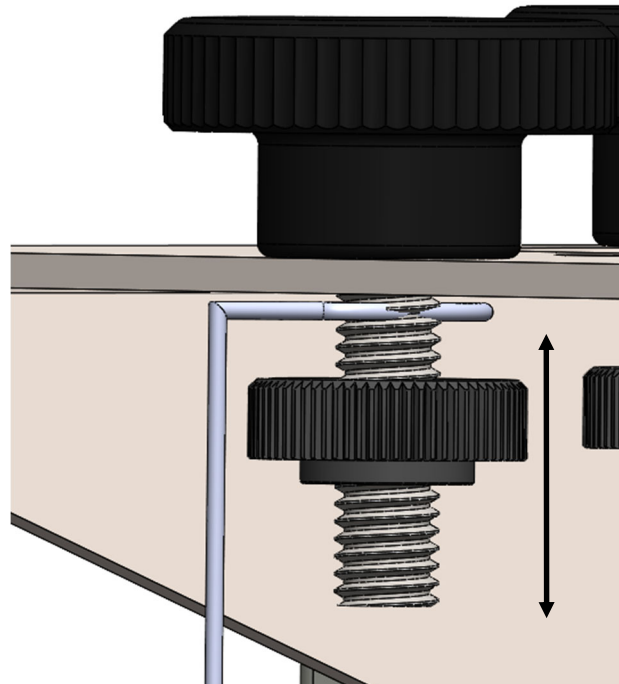


### Over to you:

A simple pendulum consists of a point mass hanging from a inextensible string of negligible mass. The effect of air resistance on it as it moves can be ignored.

Set up the system shown in the diagram, with the baseboard mounted in 'portrait' orientation.

- Connect a pendulum cord to either side where the guide rod holes are positioned and sandwich the spring in as shown.
- Hang a full (500g) hanger through the loop at the bottom of the cord.



Tighten the string with the thumbscrew

# Worksheet 8

## The simple pendulum

### Over to you.....

- Measure the length, **L**, of the pendulum. This length is the distance from the support at the top to the centre of gravity of the pendulum 'bob'. Assume that this centre of gravity is half-way down the set of weights on the hanger.
- By touching the hanger gently, steady it so that it neither rotates nor swings.
- Now, push the 'bob' to one side, a distance of about 20mm, and release it, again steadying it to reduce any rotation.
- Using a stopwatch, time 25 oscillations of the pendulum.
- Divide this time by 25 to calculate the period, **T**, of the oscillation.
- Record these readings in the Student Handout.
- Now, hang a second full (500g) hanger through the loop at the bottom of the cord. This doubles the mass of the 'bob' without having a significant effect on the position of its centre of gravity (and hence the length of the pendulum.)
- Repeat the procedure to measure the oscillation period for the heavier pendulum.
- Compare the two values for period **T** and comment on this comparison in the Student Handout.
- Next, set up another pendulum in the same way but using a longer cord.
- Measure the period of oscillation for the lighter (500g) bob and record it in the Student Handout.
- Compare the values of period **T** for the long and short pendulums and comment on this comparison in the Student Handout.

### So what:

- Calculate the theoretical period, **T<sub>c</sub>**, of the two pendulums using the following formula:

$$T_c = 2\pi \sqrt{\frac{L}{g}}$$

where **g** = gravitational field strength.

- Record the results in the Student Handout.

# Worksheet 9

## Kater's pendulum

Historically, accurate timepieces relied ultimately on the behaviour of the pendulum. However, its period was found to depend on the strength of the Earth's gravitational field in that location.

Kater's pendulum was developed to measure the strength of this field at various places around the Earth. It has the advantage that its centre of gravity and oscillation do not have to be determined.



### Over to you:

Kater's pendulum is a reversible pendulum, consisting of a rigid rod with two pivot points attached, one near each end. An adjustable weight can be moved up and down the rod to adjust the period of oscillation.

It is set swinging from one pivot and its oscillation period is determined.

It is then turned upside down and swung from the other pivot and its new period determined.

The adjustable weight is moved until these two periods are equal.

At this point this period is equal to the period of an 'ideal' simple pendulum of length equal to the distance between the pivots. From this period and the measured distance between the pivots, the acceleration of gravity can be calculated accurately.

Set up the system as shown in the diagram opposite, with the baseboard mounted in 'portrait' orientation.



# Worksheet 9

## Kater's pendulum

### Over to you.....

- Position the adjustable weight at a distance of 20mm from the pivot labelled A in the diagram overleaf.
- Suspend the pendulum from pivot A.
- Set it swinging with an amplitude of ~ 50mm, as measured at the bottom of the rod.
- Time 25 oscillations and hence calculate the period, **T**, of the oscillation.
- Record these readings in the Student Handout.
- Calculate the effective length, **L**, of the pendulum using the formula:

$$L = \frac{L^2 g}{4\pi^2}$$

- Move pivot B so that the distance between the pivots matches this length.  
(This will change the period of the pendulum slightly, but repeating this procedure once or twice will arrive at stable values of effective pendulum length and period.)
- Now suspend the pendulum from pivot B.
- Determine the period of oscillation, using the same procedure as before.
- Record the readings in the Student Handout.
- Move the adjustable weight until it is in contact with pivot A.
- Measure the period of oscillation in both pendulum orientations.
- Record these values in the Student Handout.
- Move the adjustable weight down the rod in 5mm increments, measuring the period of oscillation each time for each orientation.
- When around 20 to 30mm past the weight's original position continue in 10mm increments.



# Worksheet 9

## Kater's pendulum

### So what:

- Use your results to plot graphs of oscillation period vs distance of the adjustable weight from pivot A for the two orientations of pendulum.
- Draw smooth curves through your experimental points.
- The curves cross where the period is the same for both orientations for that distance. Now repeat the experiment to determine this intersection more accurately. To do so, set the adjustable weight a distance of 5mm above the first value for this intersection and work across it in 1mm increments, to 5mm below it, determining the oscillation period each time.
- Record all readings in the Student Handout.
- Plot the graph of oscillation period vs distance from pivot A again and use it to establish the intersection point more accurately.
- Use this graph to determine the period,  $T$ , at the intersection point and the effective length,  $L$  of the pendulum.
- Determine the local value of the acceleration due to gravity,  $g$ , using the formula:

$$g = \frac{4\pi^2 L}{T^2}$$

- Record your answer in the Student Handout.

### Challenge:

In the Student Handout, describe possible sources of error in this investigation and suggest ways of reducing them.

# Worksheet 10

## The trifilar pendulum

The pendulum has other uses apart from time-keeping. The trifilar pendulum, (= three cords,) allows us to measure the moment of inertia of a rotating body, such as a flywheel.

**Inertia** measures a body's resistance to changes in its **linear** motion. It determines the **force** needed to give it a particular acceleration.

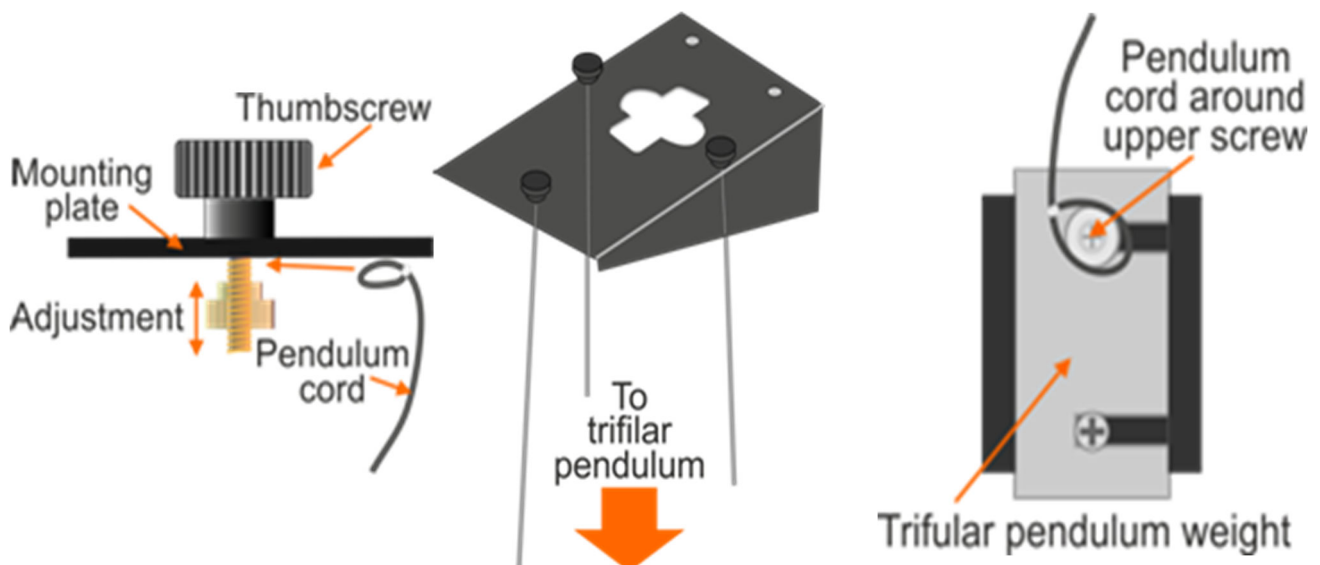
Its **moment of inertia** measures resistance to changes in **rotational** motion and determines how much **torque** is needed to give it a particular rotational acceleration.



### Over to you:

Set up the system shown in the diagram opposite, with the baseboard mounted in 'portrait' orientation.

- Connect the tops of the shorter three trifilar pendulum cords to the thumbscrews on the mounting plates, as shown opposite.
- Loop the bottoms of these pendulum cords around the upper screws on the trifilar pendulum weights.
- Adjust the heights of the three nuts on the thumbscrews until the trifilar pendulum weight is horizontal.



# Worksheet 10

## The trifilar pendulum

### Over to you.....

- Measure:
  - the pendulum 'bob' radius, **r**, i.e. from its centre to one of the supporting cords in m;
  - the mass, **m**, of the pendulum 'bob' in kg;
  - the length, **L**, of a supporting cord (from the top suspension point to half-way down the pendulum 'bob') in m.
- Turn the bob through a few degrees, (so that one of the weights moves sideways around 20mm,) and release it.
- The 'bob' now twists backwards and forwards. Try to reduce any unwanted sideways movement by touching the 'bob' very gently.
- Using a stopwatch, time 25 oscillations.
- Divide this time by 25 to calculate the period, **T**, of the oscillation.
- Record these readings in the Student Handout.
- Next, set up the trifilar pendulum in the same way but using the longer cords.
- Measure the new length, **L** and enter it in the Student Handout.
- Measure the new period of oscillation, **T** and record it in the Student Handout.
- Compare the values of period **T** for the long and short pendulums and comment on this comparison in the Student Handout.

### So what:

- Calculate the moments of inertia, **I**, of the two pendulums using the following formula:

$$I = \frac{mgr^2T^2}{4\pi^2L}$$

where **g** = gravitational field strength.

- Record the results in the Student Handout.

### Challenge:

- Investigate the effect on the oscillation period, **T**, of increasing the size of the initial deflection. What effect does this have on the measured value of moment of inertia? Comment on your findings in the Student Handout.

# Chapter 3

## Centrifugal Forces

When a body is moving in a circle, there seems to be a force pushing it away from the centre of that circle. This force, known as the centrifugal force, is not a real force in the sense that we regard gravitational, electrical and magnetic forces as real. Rather, it is the result of inertia - the tendency of an object to resist any change in its state of motion.

### Centrifugal versus centripetal:

**Centrifugal** = 'fleeing from the centre'

**Centripetal** = 'seeking the centre'

To make an object move in a circle, a force must be applied to pull it towards the centre of the circle. That force is known as the centripetal force.

For example, the Moon stays in orbit around the Earth because of the gravitational force pulling it towards us. In this case, gravity provides the centripetal force.



Thrill-seekers at a fairground are whirled round in circles because of the tension in the cables attached to their seats. Here, this tension provides the centripetal force.



The people on this ride will feel as if they are being thrown outwards by a 'centrifugal force'. What is actually happening is that their inertia is trying to keep them travelling in a straight line.

Bodies weigh more at the Earth's poles than at the equator, where part of the weight pulls the body around the Earth.

Cars slide when turning tight corners at speed.

### Applications:

- The centrifuge - used in medical applications to separate fluids based on their density.
  - The spin drier - wet washing whirled round at high speed drives out the water.
  - The centrifugal clutch - weighted arms swing out and press against friction pads inside the drum.
- and many more ...

# Chapter 3

## Centrifugal Forces

### Some more consequences:

- Cars slide when turning tight corners at speed. Friction between their tyres and the road is not great enough to force them around the corner. The car's inertia keeps it going in a straight line.
- People weigh more at the Earth's poles than at the equator.

Standing at the equator, their bodies are actually travelling in a circle around the Earth at over  $1600\text{km}\cdot\text{h}^{-1}$ . To do so, part of their weight is pulling them around the Earth.

### Uses for the centrifuge:

1. In medicine, to separate components of different density in liquid samples.
2. To prepare astronauts for the extreme forces they will experience in space.
3. To test models of structures such as bridges by exposing them to a range of forces.
4. In the home, to dry wet washing (a.k.a. spin drier!)

### A common experience -

You are standing in a moving bus.



It stops and a force pushes you forwards!

**WRONG!**

You carried on moving forwards because of your **inertia**.

The bus stopped because its brakes applied a frictional force to stop it.

It turns a corner to the left and centrifugal force throws you to the right-hand side!

**WRONG!**

You continued to move straight ahead because of your **inertia**.

Friction in its tyres forced the bus to turn left.

# Chapter 3

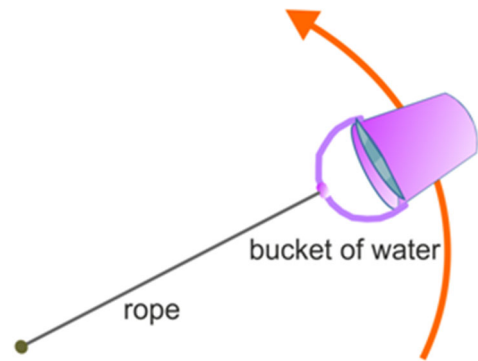
## Centrifugal Forces

### A challenge for the brave (or foolish!):

A bucket full of water can be whirled in a vertical circle around your head without the water pouring out, because its inertia keeps it pressed to the sides of the bucket.

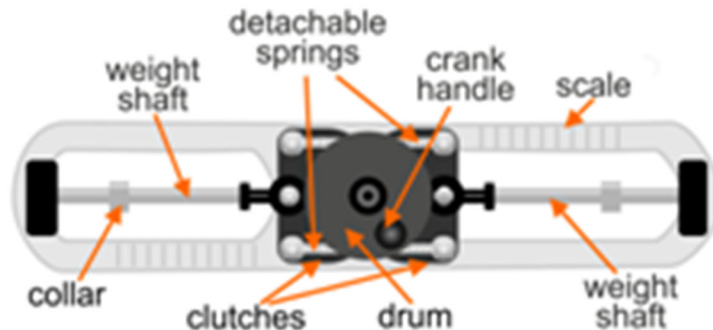
Advice:

- Use a small, toy bucket.
- Leave the practical demonstration of this to a friend!



### The experimental centrifuge:

At the heart of the device is a drum, which can be rotated by a crank handle. On either side are clutches, which can press against and restrain the drum. Springs can be added so that the clutches clamp the drum.



When the drum is rotated, a centrifugal force acts to twist the clutches away from the drum, reducing friction between the clutches and the drum.

When rotated fast enough, friction is reduced to the point where slipping occurs between the clutches and the drum.

The centrifugal force can be increased by adding weights to the weight shafts.

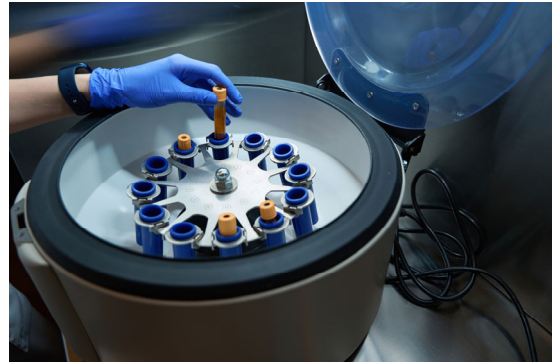
This module investigates the effect of changing the size and position of these weights to obtain the relationship for centripetal force.

# Worksheet 11

## Preliminary Centrifugal tests

A medical centrifuge uses centrifugal force to separate mixtures based on their density. Samples are placed in tubes within the rotor. When it is spun at high speed, denser components move outwards, leaving less dense components closer to the centre.

This principle allows for efficient separation of components in a liquid sample, such as different types of cells in a blood sample.

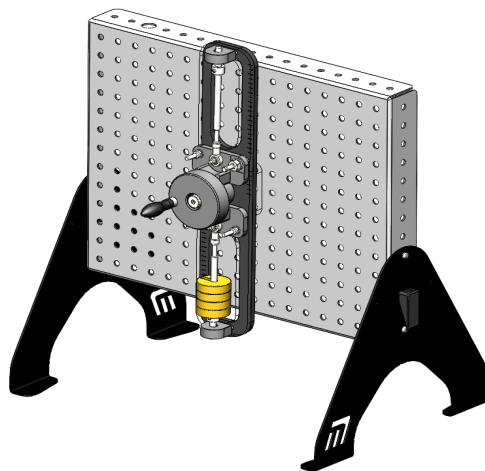


### Over to you:

#### A. Measuring spring tension:

The first task uses gravity to pivot the clutch away from the central drum in order to estimate the force needed.

- Set up the system shown in the diagram, with the baseboard mounted in 'portrait' orientation and the experimental centrifuge clamped to it in a central position.



- Attach a single spring to the lower clutch (but not the top clutch). This spring presses the lower clutch against the drum.
- Add a 50g slotted mass to sit on the collar on the lower weight shaft.
- Test whether the clutch has twisted away from the drum by slowly rotating the drum backwards and forwards a little using the crank handle. When the load is big enough, friction will be so small that it has little effect on the body of the centrifuge.
- Add further weights, one at a time, until you find the weight needed to allow slip between the clutch and the drum.
- Record this value of load in the Student Handout.

# Worksheet 11

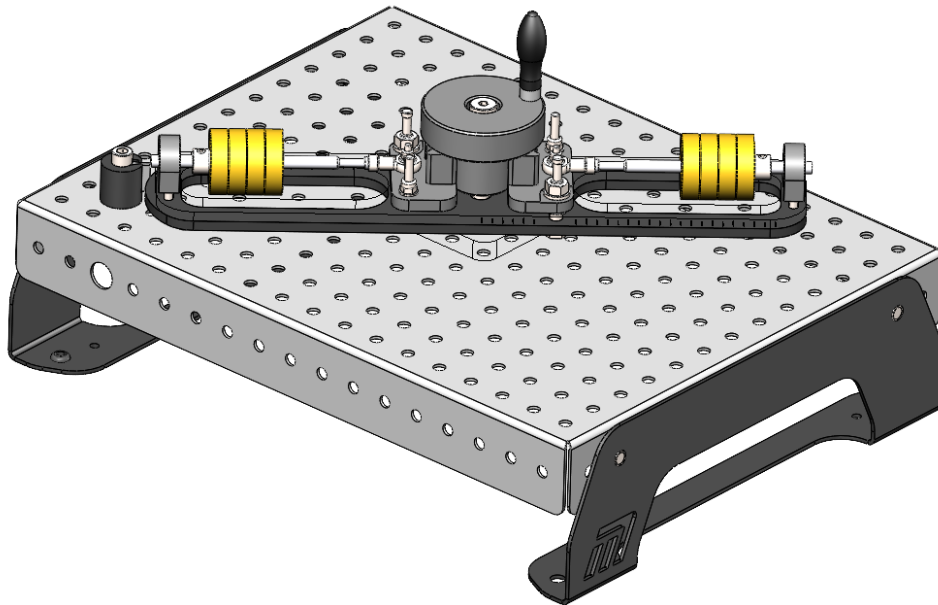
## Preliminary Centrifugal tests

Over to you.....

### B. Initial configuration:

The next task demonstrates that the inertia of rotating masses will push them outwards.

- Set up the system shown below, with the baseboard mounted horizontally and the experimental centrifuge clamped to it in a central position.



- Move the collars to their extreme position, as far away as possible from the drum.
- Turn the crank handle. Nothing happens to the centrifuge as there is no force pressing the clutches onto the drum.
- Attach four springs to the centrifuge. These press the clutch against the drum.
- Now try turning the crank handle.

Now there is a frictional force between clutches and drum.

- Add a 50g load to each weight shafts, placing them as close to the drum as possible.
- Now twist the crank handle, slowly at first and then faster. The loads should slide outwards. The faster you spin them the quicker they travel outward.

**(Note: Do not spin the centrifuge too fast! This could cause damage or injury.)**



# Worksheet 12

## Measuring centrifugal force

A **centrifugal clutch** uses centrifugal force to operate automatically when the engine reaches a certain speed.

As engine speed increases, weighted arms in the clutch swing outwards and press against friction pads inside a drum, forcing the clutch to engage.

It is often used in small-engine vehicles such as mopeds and lawn mowers.



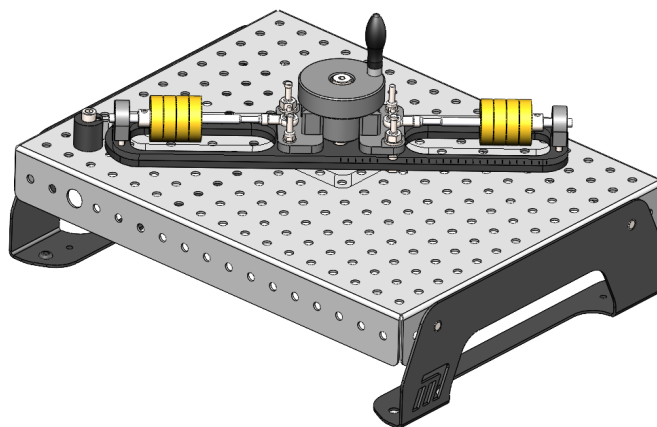
### Over to you:

The task now is to estimate the centrifugal force produced in a number of scenarios:

- using different numbers of springs, to change the force needed to release the drum from the clutches;
- changing the number and positions of the weights to change the centrifugal force resulting from a particular speed of rotation.

To do so, students try to detect the point at which the frictional force between the clutches and the drum drop to zero so that the arms of the centrifuge no longer accelerate.

At this point, the centrifugal force on the load equals the force of the spring on the load, (measured in worksheet 1.)



- The first scenario, 'setup **a**', is described in the table in the Student Handout. It involves having four springs attached to the centrifuge and adding a total of 200g to each weight shaft. Assemble these on the experimental centrifuge.
- Check that the collars are still at their extreme position, as far away as possible from the drum. This means that the centres of gravity of the weights sit at a distance of 120mm to give the required radius of rotation,  $r$ .

# Worksheet 12

## Measuring centrifugal force

### Over to you.....

- Now spin the centrifuge by turning the drum, slowly at first and then building up until a maximum speed has been reached. The centrifuge won't spin any faster because the clutches have lost their grip on the drum. (You will be able to feel when this happens.)
- Continue to spin the centrifuge at this speed and start timing on a stopwatch.
- Count the number of 'clicks' made by the rotor in 15 seconds.
- Record this result in the table in the Student Handout.
  
- Now build 'setup **b**', described in the table in the Student Handout.
- Once again, count the number of 'clicks' in 15s once the centrifuge reaches its maximum speed.
  
- Repeating the same process, set up and test the other scenarios described in the table.

### So what:

- In 15s, **n** 'clicks' are produced.
- Multiply 'n' by 4 to obtain revolutions per minute.
- You need the 'w' in  $\text{rad.s}^{-1}$ . Do this by taking your revolution per minute and multiplying it by  $\frac{\pi}{180}$

$$\frac{\pi}{180}$$

- 1 revolution =  $w = 2\pi f = \frac{2\pi}{T}$

- Centrifugal force,  $F_c$ , is calculated from the formula:

$$F = mw^2r$$

When **m** is in kg and **r** is in mm,  $F_c$  is in mN. (where 1N = 1000mN)

- Use these formulae to calculate the angular velocity and centrifugal force for each of the scenarios **a** to **h** and complete the table in the Student Handout with the results.

# Worksheet 12

## Measuring centrifugal force

### So what.....

- We already have values for the centrifugal force for each scenario.
- Worksheet 1 measured the force needed to overcome friction between clutch and drum (and hence achieve maximum rotational speed) when one spring was used on each clutch.
- In those scenarios where two springs were used on each clutch, this value is doubled. (The effect of combining springs in parallel was explored in another module, “Springs”.)

### Challenge:

- Compare the theoretical and experimental values for centrifugal force.
- In the Student Handout, comment on this comparison.
- Then suggest possible sources of error in this investigation and ways of reducing them.

# Student Handout

**Worksheet 1 - Single compression spring**

**Challenge:**

Spring colour - Silver			
Load in g	Load in N	Scale reading in mm	Deflection in mm
0	0		0
60			
120			
180			
240			
300			
360			
420			
480			
Spring constant =			
Spring colour - White			
Load in g	Load in N	Scale reading in mm	Deflection in mm
0	0		0
60			
120			
180			
240			
300			
360			
420			
480			
Spring constant =			

Spring colour - Red			
Load in g	Load in N	Scale reading in mm	Deflection in mm
0	0		0
60			
120			
180			
240			
300			
360			
420			
480			
Spring constant =			
Spring colour - Blue			
Load in g	Load in N	Scale reading in mm	Deflection in mm
0	0		0
60			
120			
180			
240			
300			
360			
420			
480			
Spring constant =			

Identify possible sources of error and suggest ways to reduce these:

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**Worksheet 2 - Compression springs in parallel**

**Challenge:**

Spring colour - Silver			
Load in g	Load in N	Scale reading in mm	Deflection in mm
0	0		0
60			
120			
180			
240			
300			
360			
420			
480			
Spring constant =			
Spring colour - White			
Load in g	Load in N	Scale reading in mm	Deflection in mm
0	0		0
60			
120			
180			
240			
300			
360			
420			
480			
Spring constant =			

Spring colour - Red			
Load in g	Load in N	Scale reading in mm	Deflection in mm
0	0		0
60			
120			
180			
240			
300			
360			
420			
480			
Spring constant =			
Spring colour - Blue			
Load in g	Load in N	Scale reading in mm	Deflection in mm
0	0		0
60			
120			
180			
240			
300			
360			
420			
480			
Spring constant =			

Comment on the performance of this system compared to that in worksheet 1:

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**Worksheet 3 - Compression springs in series**

**Challenge:**

Spring colour - Silver			
Load in g	Load in N	Scale reading in mm	Deflection in mm
0	0		0
60			
120			
180			
240			
300			
360			
420			
480			
Spring constant =			
Spring colour - White			
Load in g	Load in N	Scale reading in mm	Deflection in mm
0	0		0
60			
120			
180			
240			
300			
360			
420			
480			
Spring constant =			

Spring colour - Red			
Load in g	Load in N	Scale reading in mm	Deflection in mm
0	0		0
60			
120			
180			
240			
300			
360			
420			
480			
Spring constant =			
Spring colour - Blue			
Load in g	Load in N	Scale reading in mm	Deflection in mm
0	0		0
60			
120			
180			
240			
300			
360			
420			
480			
Spring constant =			

Comment on the performance of this system compared to that in worksheet 1:

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## Worksheet 4 - Single extension spring

Load in g	Load in N	Scale reading in mm	Deflection in mm
0	0		0
100			
200			
300			
400			
500			
<b>Spring constant =</b>			

**Challenge:**

Explain what would happen if two force meters were hung, one below the other and a weight suspended from the lower one.

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**Worksheet 5 - Extension springs in series**

Load in g	Load in N	Scale reading in mm	Deflection in mm
0	0		0
100			
200			
300			
400			
500			
<b>Spring constant =</b>			

**Challenge:**

Find out as much as you can about progressive (variable) rate springs:

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**Worksheet 6 - Extension springs in parallel**

Load in g	Load in N	Scale reading in mm	Deflection in mm
0	0		0
100			
200			
300			
400			
500			
<b>Spring constant =</b>			

**Challenge:**

Describe one situation where combining springs in parallel would be preferable to using a single spring.

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**Worksheet 7 - Oscillating spring**

Load in g	Load W in N	Length of spring in mm	Extension in mm	Time for 25 oscillations in s	Period T in s	Experimental estimate of mass m in g
0	0		0	0	0	
200						
300						
400						
500						
600						

Spring constant,  $k = \dots\dots\dots$

**Challenge:**

1. What did you discover about the effect of initial displacement on the period of oscillation,  $T$ ?

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2. Results for investigating a spring with a bigger spring constant:

Spring constant,  $k = \dots\dots\dots$

Load in g	Load W in N	Length of spring in mm	Extension in mm	Time for 25 oscillations in s	Period T in s	Experimental estimate of mass m in g
0	0		0	0	0	

**Worksheet 8 - The simple pendulum**

**Shorter pendulum**

Load in g	Time for 25 oscillations in s	Period T in s
500		
1000		

Length of pendulum = .....

Comment on the effect of doubling the mass of the 'bob' on the period of oscillation, **T**?

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**Longer pendulum**

Load in g	Time for 25 oscillations in s	Period T in s
500		

Length of pendulum = .....

Comment on the effect of increasing the length of the pendulum on the period of oscillation, **T**?

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**Worksheet 8 - The simple pendulum**

**So what:**

Theoretical period,  $T_c$ , of the pendulum = .....

**Challenge:**

Comment, quoting results, on the investigation into the compound pendulum:

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## Worksheet 9 - Kater's pendulum

### Initial setup - pendulum suspended from pivot A:

Distance from weight to pivot A = 20mm

Time for 25 oscillations = ..... s

Oscillation period, **T** = ..... s

Effective length  $L = \frac{T^2 \times g}{4 \times \pi^2} = \dots\dots\dots$  mm

### Pendulum suspended from pivot B:

Time for 25 oscillations = ..... s

Oscillation period, **T** = ..... s

Effective length **L** = ..... mm

### Results for adjustable weight re-positioning

Distance from weight to pivot A in mm	Suspended from A		Suspended from B	
	Time for 25 oscillations in s	Period T in s	Time for 25 oscillations in s	Period T in s
0				

## Worksheet 10 - The trifilar pendulum

### Shorter pendulum

Length, **L**, of cord = .....

Mass, **m**, of pendulum 'bob' = .....

Pendulum 'bob' radius, **r** = .....

Time for 25 oscillations = .....

Period, **T**, of oscillation = .....

Moment of inertia, **I** = .....

### Longer pendulum

Length, **L**, of cord = .....

Mass, **m**, of pendulum 'bob' = .....

Pendulum 'bob' radius, **r** = .....

Time for 25 oscillations = .....

Period, **T**, of oscillation = .....

Moment of inertia, **I** = .....

### Challenge:

Investigate the effect on the oscillation period, **T**, of increasing the size of the initial deflection.

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**Worksheet 10 - The trifilar pendulum**

Results for accurate location of intersection point

Distance from estimated intersection in mm	Suspended from A		Suspended from B	
	Time for 25 oscillations in s	Period T in s	Time for 25 oscillations in s	Period T in s
5				
4				
3				
2				
1				
0				
-1				
-2				
-3				
-4				
-5				

At the point of intersection:

effective length of pendulum, **L** = ..... mm

period of oscillation, **T** = ..... s

Using these:

local value of the acceleration due to gravity,  $g = \frac{4 \times \pi^2 \times L}{T^2} = \dots\dots\dots \text{m.s}^{-2}$

**Challenge:**

Possible sources of error:

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**Worksheet 11 - Preliminary Centrifugal tests**

**A. Measuring spring tension:**

Mass of load needed to release drum = .....g

Weight of this load = .....N

**Worksheet 12 - Measuring centrifugal force**

Setup	No. of springs	Mass m in kg	Radius r in mm	No. of 'clicks' n in 15s	Angular velocity of load $\omega$ in $\text{rad.s}^{-1}$	Experimental centrifugal force in mN
<i>a</i>	4	0.2	120			
<i>b</i>	4	0.2	100			
<i>c</i>	4	0.1	120			
<i>d</i>	4	0.1	100			
<i>e</i>	4	0.1	80			
<i>f</i>	2	0.1	120			
<i>g</i>	2	0.1	100			
<i>h</i>	2	0.1	80			

**Challenge:**

Comparison of theoretical and experimental results:

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Possible sources of error and ways to reduce them:

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