MATRIX | MATERIALS

CRITICAL CONDITIONS OF STRUTS

CP4179

Inspiring The Next Generation Of Engineers

www.matrixtsl.com

Contents

Laboratory Technique

Safety in the Laboratory

The principal hazards in using apparatus that demonstrates the static and dynamic performance of associated theorems and the assumptions involved are where rotary or linear motion occurs and where the handling of loose heavy items, for example weights, is part of the procedure.

Of the loose items the heavier weights must be regarded as the most dangerous objects. Should one of these fall onto the feet of those around the apparatus the potential for damage is present. Hence it is recommended that cast iron weights be handled carefully and when moving and placing the heavier ones (say 10 N upward) on load hangers this should be regarded as a two handed operation. It is surprisingly easy to spill a complete stack of weights off a hanger when adding a further one.

In addition to weights there are some heavy parts that have to be interchanged during some experiments and a similar approach using two hands where required is suggested. It may also be both sensible and necessary for two people to take part in changes to the apparatus.

Success in the Laboratory

Work in the laboratory depends on understanding, observation and skill. In the first place a good understanding of the performance, and limitations, of experimental models is needed. To know about the theory involved is useful but not essential. In the second place keen observation leads to better results and avoidance of mechanical mistakes. Lastly, the way in which students handle the apparatus can influence the accuracy and speed of the work.

To help students gain experience and improve their experimental technique a range of information is offered in the following notes. Bear in mind that in the world of real engineering it is often necessary to check the performance of new designs using the methods and instruments of laboratory experiments.

Design of Experimental Models

The purpose of each experiment is to illustrate an item of structural theory, or to show how well simplifying assumptions in the applied mathematics correspond to actual behaviour. This often requires the model to exaggerate the behaviour of a real structure.

In order to achieve specific objectives each experiment has a particular arrangement best suited to the theoretical requirement. These arrangements of the apparatus are described in the Construction Appendix, where included, of each experimental Instruction Manual. Before starting an experiment students should read through the Instruction Manual and be prepared to follow the recommended procedure.

Laboratory Technique

Increased deflections are usually achieved by using very flexible models. The stiffness depends on ***EI*** or ***EA*** so a change of material from steel (**E** = 205 kN/mm2) to aluminium (**E** about 1/3 **E** for steel) or a plastic (**E** about 1/80 **E** for steel) is a solution. The alternative is to use thin steel beams with a low ***I***.

One disadvantage experimentally is that friction in bearings may affect displacements and force measurements. The other is that large changes in dimension (geometry) of models must be accommodated if possible.

Results can be improved by using stiffer models and larger loads, but this reduces visual effects such as curvature of beams.

Sources of Resistance

A knife-edge can simulate a frictionless pin or bearing, but horizontal and rotational movements demand ball bearings. These are packed with grease and fitted with shields to keep out dust and grit. Hence ball bearings have some torsional restraint, which affects forces in the order of magnitude 1 N. This shows up as a difference in readings for loading and unloading.

Pin joints in trusses are also subject to friction, which increases in proportion to the loading.

Repeatability of Readings

The ability to obtain accurate and repeatable experimental results is generally a matter of care and technique. Of course it helps to know the sources of error and to recognise when the apparatus contributes to the variability of readings.

Frictional variation can be minimised by using vibration. The extent of the friction can be observed by first increasing and then decreasing an applied load by hand to get the difference in readings. Tapping the frame on which the experiment is mounted will reduce the variation.

Cast iron weights for loading must always be applied gently. A load suddenly added will instantaneously apply twice its static value. Although weights are hand finished there is a manufacturing tolerance of ± ½%. This may affect linearity in experimental readings.

Introduction

The failure mechanism of a compression member is more often than not that of buckling. It is an example of instability which usually results in ultimate collapse of the structure. There are two theoretical approaches to the study of struts. The first is the classical mathematical elastic theory of Euler based on a perfectly straight uniform strut. The second is the strength of materials theory applied to a real strut with slight imperfections where failure occurs when the stress reaches the yield or proof stress value.

This apparatus shows how the buckling mechanism occurs, and how it is affected by the end conditions of the strut. For simplicity the Euler theory is used as the mathematical model.

APPARATUS ASSEMBLY

Apparatus Description

The unit comes fully assembled with all accessories required. The only assembly set up required is to screw the M5 x 60 length of studding into the D position Loading Pad other than preparing the unit for the first experiment. This is outlined in the procedures section of this manual.

The apparatus comprises a sheet metal frame which supports four struts with loading platforms at their top ends. The struts, made of spring steel, are 180 mm long and have a cross section 9.53 x 0.56 mm. The end conditions are:-

A - Both ends pinned (strip can rotate)
B - One end pinned, one end fixed
C - Both ends fixed
D - Base fixed, top free (flagpole)

In A, B, and C the ends can move inward as the strut buckles. The loading acts through guide bushes which should be frictionless to match the theory.

For strut D the loading platform is guided in a slot which permits the top of the strut to sway sideways. Strictly the load should be 180 mm above the fixed base.

EXPERIMENT

Objectives / Procedure

Objectives

The purpose of the work is to compare the buckling loads of the four struts, with Euler's mathematical critical loads namely

A (pinned) $\quad P = \dfrac{\pi^2 EI}{L^2}$

B (fixed/pinned) $\quad P = 2.045\,\dfrac{\pi^2 EI}{L^2}$

C (fixed) $\quad P = \dfrac{4\pi^2 EI}{L^2}$

D (fixed/free) $\quad P = 0.25\,\dfrac{\pi^2 EI}{L^2}$

Procedure

Calculate the Euler critical load for strut A using E = 205 kN/mm2. (The nominal value is about 8.5 N)

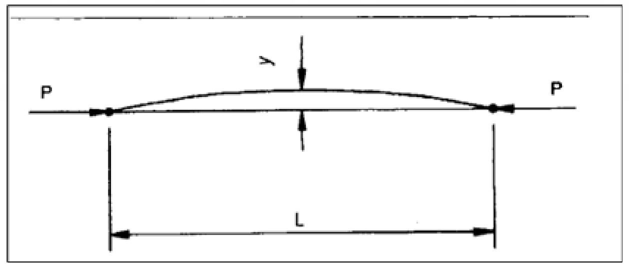
Place 8 N on strut A and gently push the middle of the strut sideway by two or three mm. If the strut does not suddenly buckle then the load is below the critical value. On removing one's finger the strut becomes straight. Repeat this test with increasing loads until the strut buckles when touched.

Record the critical load. Sketch the deflected shape of the strut.

Factor the critical load just found to predict the critical values for the other three struts and repeat the above procedure. It will be found that strut D is prone to instability because the load is higher than it should be. For strut D, due to its instability (and to a certain extent strut C), the loading plate has a tapped thread for adding a rod to help with keeping weights central.

THEORY
Euler's Theory

Euler's Theory



Euler's basic strut was a pin ended column length L with perfect axial loading P. If such a strut should deflect transversely then a bending moment Py is imposed where y (+ve in the diagram) is the deflection due to that bending moment. With the sign convention adopted it can be seen that the moment is -ve because the strut is hogging. Then by substitution in we get;

$$EI\frac{d^2y}{dx^2} = M$$

$$\frac{d^2y}{dx^2} = \frac{-Py}{EI}$$

and writing a^2 for $\frac{P}{EI}$

The solution of this differential equation is;

$$y = A\cos ax + B\sin ax$$

Since Y = 0 when x = 0 and L, A = 0 and 0 = B sin aL.

This equality is satisfied if: - B = 0
Or if: - sin aL = 0

If B = 0 there is no deflection anywhere and hence no bending moment. Suppose that for any reason there is some deflection. Then from sin aL = 0 it follows that

AL = 0 or π 0r higher multiples

If aL = 0 then P = 0 Which is again not real. Hence the real solution is;

aL = π

from which; $$P = \frac{\pi^2 EI}{L^2}$$

RESULTS
Results Processing

Compare the theoretical and actual critical loads for the four struts.

An alternative way of deducing the critical loads of struts B, C and D is to assess how the buckled shape of A can be overlaid on a portion of the buckled shapes of B and C, or how D fits against A. As P is inversely proportional to L2, show how this lines up with the ratios of the critical loads.

Observations

Bearing in mind that Euler's conditions were a perfectly straight and uniform strut exactly axially loaded, did the set of strut results verify the theory?

In what way will the imperfections of the apparatus influence the results?

RESULTS
Example Results

The following results were obtained by an experienced demonstrator.

Strut	Critical Load (N)	Ratio to A
A	8.5	1.0
B	15.0	1.8
C	34.0	4.0
D	2.5	0.3

The following pictures show the different buckling of the four main setups, the weights however have not been added under test conditions so the amount of weight on each strut may not reflect the appropriate result finding.

For strut D, due to its instability (and to a certain extent strut C), the loading plate has a tapped thread for adding a rod to help with keeping weights central. Smaller weights can also be used (compared to the images above) to allow for more accurate results and easier to balance loads.

RESULTS
Example Results

CRITICAL CONDITIONS OF STRUTS

CP4179

www.matrixtsl.com

Matrix Technology Solutions
The Factory, 33 Gibbet Street, Halifax, HX1 5BA, United Kingdom
t: +44 (0) 1422 252380   e: sales@matrixtsl.com